

Acknowledgements	ix
<b>1 Introduction</b>	<b>1</b>
1.1 The Classical Model of Science and the notion of explanation in axiomatics . . . . .	3
1.2 Grounding – the general idea . . . . .	6
1.2.1 Three cases . . . . .	6
1.2.2 Six claims . . . . .	13
1.3 Overview of the literature . . . . .	15
1.4 What I will not do in this thesis . . . . .	17
1.5 Principles of interpretation . . . . .	18
1.6 Overview of the thesis . . . . .	19
<b>2 The <i>Beyträge</i>: Bolzano’s early views on grounding</b>	<b>25</b>
2.1 Introduction . . . . .	25
2.2 Preparations . . . . .	29
2.2.1 Scope and historical context of the <i>Beyträge</i> . . . . .	29
2.2.2 Scientific presentation and scientific proofs . . . . .	31
2.2.3 The logic of the <i>Beyträge</i> . . . . .	34
2.3 Ways of being objectively dependent and criteria of axiomhood . . . . .	45
2.3.1 In search for an objective notion of axiomhood . . . . .	46
2.3.2 A set of rules . . . . .	48
2.3.3 Other rules? . . . . .	54
2.3.4 A necessary condition for axiomhood . . . . .	56
2.4 Desiderata for presentations and proofs . . . . .	61
2.4.1 (D <sub>1</sub> ) & (D <sub>2</sub> ): Simplicity and generality . . . . .	62
2.4.1.1 (D <sub>2</sub> ): The Complexity Constraint in the BD . . . . .	63

2.4.1.2	(D <sub>1</sub> ): The Generality Constraint in the BD . . .	65
2.4.2	(D <sub>3</sub> ): Relevance, redundancy and the fallacy of kind-crossing	68
2.4.3	(D <sub>4</sub> ): Uniqueness . . . . .	73
2.4.4	Wrapping up . . . . .	78
2.5	The <i>Beyträge</i> 's conception of science . . . . .	79
2.6	Conclusion . . . . .	83
2.7	(Appendix) Varieties of dependence: Bolzano's <i>Aetiologie</i> . . . . .	84
2.7.1	Some examples . . . . .	85
2.7.2	Axioms of <i>Aetiologie</i> . . . . .	87
2.7.3	<i>Aetiologie</i> and explanatory dependence . . . . .	88
<b>3</b>	<b>Objective truth, variation, truth-preservation</b>	<b>91</b>
3.1	Introduction . . . . .	91
3.2	The basic framework of the WL . . . . .	93
3.2.1	Propositions and ideas . . . . .	94
3.2.2	Concepts and intuitions . . . . .	98
3.2.3	Form and truth . . . . .	101
3.3	The logic of variation . . . . .	110
3.3.1	Remarks on collections and notation . . . . .	110
3.3.2	The definition of a variant . . . . .	112
3.3.3	Analyticity in the WL . . . . .	115
3.3.4	Deducibility . . . . .	119
3.3.4.1	Basic properties of deducibility . . . . .	122
3.3.4.2	Logical deducibility . . . . .	123
3.3.4.3	Equivalence . . . . .	124
3.3.5	Exact deducibility . . . . .	125
3.4	Conclusion . . . . .	130
<b>4</b>	<b>Explanatory priority: Bolzano's pure logic of grounding</b>	<b>133</b>
4.1	Introduction . . . . .	133
4.2	Preliminaries . . . . .	135
4.2.1	Existing reconstructions of Bolzano's theory . . . . .	136
4.2.2	A remark on Bolzano's method of clarification . . . . .	137
4.2.3	Grounding, deducibility and causality . . . . .	138
4.3	Some basic distinctions . . . . .	140
4.4	Immediate grounding . . . . .	146
4.4.1	Irreflexivity, Aristotle's insight and uniqueness . . . . .	146
4.4.2	Interlude: a stronger uniqueness requirement . . . . .	153
4.4.3	Further properties of immediate grounding . . . . .	159
4.5	A more concise reconstruction . . . . .	174
4.6	Mediate grounding . . . . .	179
4.6.1	"Ascending from consequences to their grounds" . . . . .	179

4.6.2	Properties of mediate grounding . . . . .	184
4.6.3	(Appendix) Infinite grounding trees . . . . .	186
<b>5</b>	<b>Simplicity and economy: Bolzano's impure logic of grounding</b>	<b>191</b>
5.1	Introduction . . . . .	191
5.2	Conceptuality, complexity and the existence of fundamental truths	195
5.2.1	Conceptuality and complexity: WL, §221.1 & §221.2 . . . .	196
5.2.2	There are fundamental (conceptual) truths: WL, §221.3 . . .	201
5.2.3	A different notion of complexity and the ideal of purity . .	204
5.3	Economy Principles . . . . .	212
5.3.1	Grounding and deducibility . . . . .	214
5.3.2	Propositional economy: WL, §221.5 . . . . .	222
5.3.3	Partial grounds are logically independent and not logically analytic . . . . .	231
5.3.4	Conceptual economy: WL, §221.4 . . . . .	237
5.3.5	A tension in Bolzano's theory of grounding . . . . .	241
5.3.5.1	The tension explained . . . . .	241
5.3.5.2	Prospects for resolving the tension . . . . .	244
5.3.5.3	Wrapping up . . . . .	247
5.3.6	Global economy: WL, §221.3 . . . . .	248
5.3.7	A principle for sciences? . . . . .	260
5.4	The Generality Constraint: WL, §221.6 . . . . .	262
5.4.1	A traditional principle rejected . . . . .	263
5.4.2	An alternative principle . . . . .	266
5.4.3	A proposal for an interpretation . . . . .	268
5.5	Jointly sufficient conditions for grounding: WL, §221.7 . . . . .	273
5.5.1	The main idea . . . . .	274
5.5.2	Interpretation of (N <sub>1</sub> )–(N <sub>3</sub> ) . . . . .	276
5.5.3	Another attempt . . . . .	279
5.5.4	Sufficient, but also necessary? . . . . .	281
5.5.5	Grounding preserved under variation . . . . .	283
5.6	Bolzano's tentative definition of grounding: WL, §221.note . . . . .	287
5.6.1	Introduction . . . . .	287
5.6.2	Interpretation of (DfG) . . . . .	289
5.6.3	Retaining local grounding . . . . .	293
5.7	Fundamental truths revisited . . . . .	295
5.8	Conclusion . . . . .	302
<b>6</b>	<b>The WL's conception of science</b>	<b>307</b>
6.1	Introduction . . . . .	307
6.2	Recapitulating the BD . . . . .	311
6.3	Sciences and their starting points . . . . .	312

6.3.1	Sciences as collections of truths . . . . .	313
6.3.2	<i>Grundsätze</i> of a science . . . . .	315
6.3.2.1	<i>Grundsätze</i> <sub>WL</sub> vs. <i>Grundsätze</i> <sub>BD</sub> I: Groundedness	319
6.3.2.2	<i>Grundsätze</i> <sub>WL</sub> vs. <i>Grundsätze</i> <sub>BD</sub> II: Remoteness .	322
6.3.2.3	<i>Grundsätze</i> <sub>WL</sub> vs. <i>Grundsätze</i> <sub>BD</sub> III: Complexity	324
6.3.3	An illustration: geometry and the definition of the notion of location . . . . .	325
6.3.4	Concluding remarks and the CMS . . . . .	328
6.4	Domain and demarcation of sciences . . . . .	329
6.4.1	The concept of a suitable science . . . . .	331
6.4.2	Hope for an objective criterion? . . . . .	335
6.4.3	The hierarchy of sciences in the WL . . . . .	337
6.5	The Presentation of a Science . . . . .	340
6.5.1	Epistemological prelude . . . . .	341
6.5.2	The main purpose of a scientific treatise . . . . .	343
6.5.3	Proofs in rigorous scientific presentations . . . . .	350
6.5.3.1	The general notion of proof . . . . .	350
6.5.3.2	Demonstrations . . . . .	355
6.5.3.3	Definitions and deductions . . . . .	363
6.6	Conclusion . . . . .	365
7	Conclusion	367
A	Immediate predecessor	375
B	Conceptions of Science	379
C	Summary in Dutch: Bolzano's notie van fundering en het Klassiek Model van Wetenschap	383
	List of abbreviations	389
	Bibliography	391