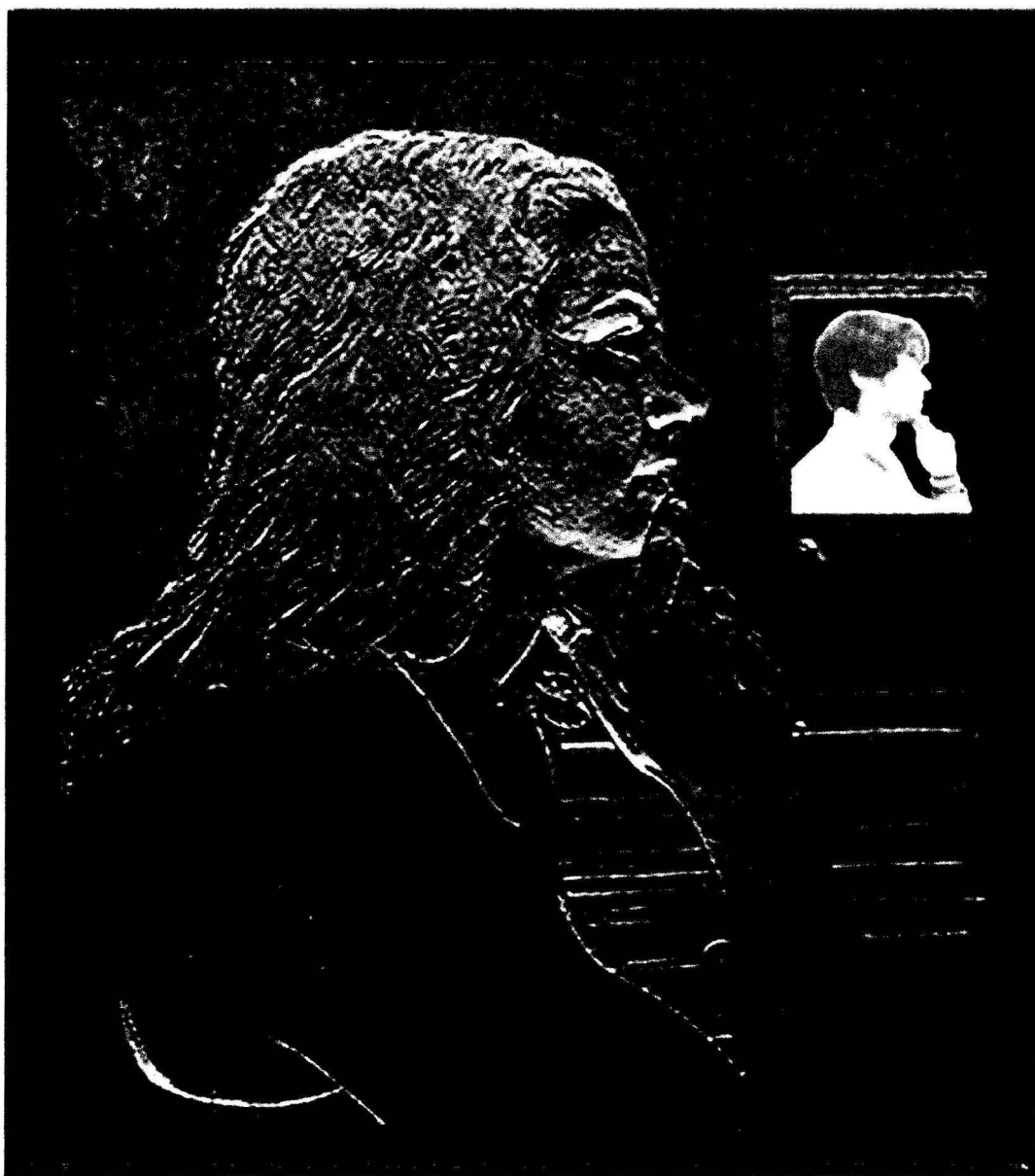


Game Theory

Volume Two: *Advanced Topics*



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