
Jürgen Jost
Editor

Bernhard Riemann
On the Hypotheses Which
Lie at the Bases of Geometry

 Birkhäuser

Contents

1	Introduction	1
2	Historical Introduction	9
2.1	The Space Problem in Physics, from Aristotle to Newton	9
2.2	Kant's Philosophy of Space	17
2.3	Euclidean Space as the Basic Model	21
2.4	The Development of Geometry: Non-Euclidian and Differential Geometry	25
2.5	The Story of Riemann's Habilitation Address	27
3	Riemann's Text	29
4	Presentation of the Text	43
4.1	Short Summary	43
4.2	The Main Results of the Text	44
4.3	Riemann's Reasoning	45
4.4	Mathematical Commentary	62
4.4.1	The Concept of a Manifold	63
4.4.2	Tensor Calculus	67
4.4.3	Metric Structures	72
4.4.4	Geodesic Curves	74
4.4.5	Normal Coordinates	79
4.4.6	Riemann's Abstract Reasoning	86
4.4.7	Flatness and Curvature	88
4.4.8	Submanifolds of Euclidean Space	101
4.4.9	Sectional Curvature	104
4.4.10	Spaces of Constant Curvature	112
4.5	Going Through Riemann's Text	115

5 Reception and Influence of Riemann's Text	119
5.1 Helmholtz	119
5.2 The Further Development of Riemannian Geometry and Einstein's Theory of Relativity	128
5.3 Lie and the Theory of Symmetry Groups	133
5.4 Weyl and the Concept of the Connection on a Manifold	134
5.5 Spaces as Tools for the Geometric Representation of Structures	136
5.6 Riemann, Helmholtz and the Neo-Kantians	137
5.7 The Axiomatic Foundation of Geometry	138
5.8 Conventionalism	142
5.9 Abstract Space Concepts	144
6 Modern Research	147
6.1 The Global Structure of Manifolds	147
6.2 Riemannian Geometry and Modern Physics	151
7 Selected Bibliography with Commentaries	155
7.1 Different Editions of the Text	155
7.2 Bibliographies	158
7.3 Introductions	158
7.4 Important Monographs and Articles	160
Glossary	163
Biographical Outline and Chronological Table	165
Index	169