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# Leibniz on the Parallel Postulate and the Foundations of Geometry

The Unpublished Manuscripts

 Birkhäuser

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