

A Student's Guide to Vectors and Tensors

DANIEL A. FLEISCH



CAMBRIDGE
UNIVERSITY PRESS

Contents

<i>Preface</i>	<i>page</i> vii
<i>Acknowledgments</i>	x
1 Vectors	1
1.1 Definitions (basic)	1
1.2 Cartesian unit vectors	5
1.3 Vector components	7
1.4 Vector addition and multiplication by a scalar	11
1.5 Non-Cartesian unit vectors	14
1.6 Basis vectors	20
1.7 Chapter 1 problems	23
2 Vector operations	25
2.1 Scalar product	25
2.2 Cross product	27
2.3 Triple scalar product	30
2.4 Triple vector product	32
2.5 Partial derivatives	35
2.6 Vectors as derivatives	41
2.7 Nabla – the del operator	43
2.8 Gradient	44
2.9 Divergence	46
2.10 Curl	50
2.11 Laplacian	54
2.12 Chapter 2 problems	60
3 Vector applications	62
3.1 Mass on an inclined plane	62
3.2 Curvilinear motion	72

3.3	The electric field	81
3.4	The magnetic field	89
3.5	Chapter 3 problems	95
4	Covariant and contravariant vector components	97
4.1	Coordinate-system transformations	97
4.2	Basis-vector transformations	105
4.3	Basis-vector vs. component transformations	109
4.4	Non-orthogonal coordinate systems	110
4.5	Dual basis vectors	113
4.6	Finding covariant and contravariant components	117
4.7	Index notation	122
4.8	Quantities that transform contravariantly	124
4.9	Quantities that transform covariantly	127
4.10	Chapter 4 problems	130
5	Higher-rank tensors	132
5.1	Definitions (advanced)	132
5.2	Covariant, contravariant, and mixed tensors	134
5.3	Tensor addition and subtraction	135
5.4	Tensor multiplication	137
5.5	Metric tensor	140
5.6	Index raising and lowering	147
5.7	Tensor derivatives and Christoffel symbols	148
5.8	Covariant differentiation	153
5.9	Vectors and one-forms	156
5.10	Chapter 5 problems	157
6	Tensor applications	159
6.1	The inertia tensor	159
6.2	The electromagnetic field tensor	171
6.3	The Riemann curvature tensor	183
6.4	Chapter 6 problems	192
	<i>Further reading</i>	194
	<i>Index</i>	195