

Enrico Martino

Intuitionistic Proof Versus Classical Truth

The Role of Brouwer's Creative Subject
in Intuitionistic Mathematics

Contents

1	Brouwer, Dummett and the Bar Theorem	1
1.1	Introduction	1
1.2	Terminology and Symbolism	1
1.3	Dummett's Argument	3
1.4	Critique of Dummett's Argument	4
1.5	Limits of the Eliminability of ζ -inferences	10
1.6	Final Considerations	12
	References	14
2	Creative Subject and Bar Theorem	15
2.1	The Creative Subject	15
2.2	The Creative Subject and Existential Statements	19
2.3	Equivalence of (PIE) and (BI_M)	21
	References	22
3	Natural Intuitionistic Semantics and Generalized Beth Semantics	23
3.1	Introduction	23
3.2	Generalized Beth-Models and Natural Models	23
3.3	Generalized Natural Models	24
	References	26
4	Connection Between the Principle of Inductive Evidence and the Bar Theorem	27
4.1	Inductive Evidence	27
	Reference	30
5	On the Brouwerian Concept of Negative Continuity	31
5.1	Introduction	31
5.2	The Negative Continuity Theorem	32
5.3	A Proof of NCP	33
5.4	Weak and Strong Negation	33

5.5	The Role of Time in Brouwer's Argument	34
5.6	Brouwer's Argument and Solipsism	35
5.7	NCP and Lawless Sequences	37
5.8	Revising NCP with the Help of the Creative Subject	40
5.9	Extensional Functions and Intensional Choice Sequences	41
5.10	Troelstra's Abstraction Process and NCP	42
5.11	Conclusions	44
	References	45
6	Classical and Intuitionistic Semantical Groundedness	47
6.1	Introduction	47
6.2	Construction of Model M	48
6.3	Axiomatisation of T	49
6.4	The Aczel–Feferman Intensional Operator	50
	References	51
7	Brouwer's Equivalence Between Virtual and Inextensible Order	53
7.1	Introduction	53
7.2	Reconstruction of Brouwer's Paper of 1927	54
7.3	Comment on Brouwer's Text	55
7.4	How Brouwer Misinterpreted Himself	57
7.5	A Minor Mistake in the Cambridge Lectures	60
7.6	On Posy's Reconstruction	61
	References	62
8	An Intuitionistic Notion of Hypothetical Truth for Which Strong Completeness Intuitionistically Holds	63
8.1	Introduction	63
8.2	Symbolism and Conventions	64
8.3	The Failure of Strong Completeness for Natural Semantics	65
8.4	Hypothetical Truth	65
8.5	Remarks on Hypothetical Truth	67
8.6	Generalized Beth Semantics	69
8.7	Connection Between Hypothetical Semantics and GB-Semantics	70
8.8	A Strong Completeness Proof for GB-Semantics	71
	References	73
9	Propositions and Judgements in Martin-Löf	75
9.1	Introduction	75
9.2	Propositions and Judgements	75
9.3	Truth and Evidence	78
9.4	Metaphysical Realism	80
	References	84

10 Negationless Intuitionism	85
10.1 Natural Semantics	85
10.2 Failure of Strong Completeness	86
10.3 Second-Order Negationless Semantics	89
10.4 Concluding Remarks	93
References	95
11 Temporal and Atemporal Truth in Intuitionistic Mathematics	97
11.1 Introduction	97
11.2 Tenselessness and Classical Truth	98
11.3 Potential Intuitionism as a Subsystem of Epistemic Mathematics	102
11.4 Temporal Truth	106
References	111
12 Arbitrary Reference in Mathematical Reasoning	113
12.1 Introduction	113
12.2 Some Objections to <i>TAR</i>	114
12.3 <i>TAR</i> as Embodied in the Logical Concept of an Object	117
12.4 The Ideal Agent	119
12.5 Arbitrary Reference and Impredicativity	122
12.6 Plural Reference Versus Sets	124
References	130
13 The Priority of Arithmetical Truth over Arithmetical Provability	133
13.1 Introduction	133
13.2 Orthodox Versus Non-orthodox Intuitionism	135
13.3 The Constructive Notion of a Process	138
13.4 Computational Realism	140
References	145
14 The Impredicativity of the Intuitionistic Meaning of Logical Constants	147
References	155
15 The Intuitionistic Meaning of Logical Constants and Fallible Models	157
15.1 Introduction	157
References	163
Origin of the Essays	165
Subject Index	167
Author Index	169