## BASIC SIMPLE TYPE THEORY

J. Roger Hindley

University of Wales, Swansea



## Contents

| Intr | oductio                       | page ix                                      |     |
|------|-------------------------------|--|-----|
| Erre | ata                           |  | xii |
| 1    | The                           | 1  |     |
|      | 1A                            | $\lambda$ -terms and their structure         | 1   |
|      | 1 <b>B</b>                    | $\beta$ -reduction and $\beta$ -normal forms | 4   |
|      | 1C                            | $\eta$ - and $\beta\eta$ -reductions         | 7   |
|      | 1D                            | Restricted $\lambda$ -terms                  | 10  |
| 2    | Assi                          | gning types to terms                         | 12  |
|      | 2A                            | The system $TA_{\lambda}$                    | 12  |
|      | 2B                            | The subject-construction theorem             | 20  |
|      | 2C                            | Subject reduction and expansion              | 24  |
|      | 2D                            | The typable terms                            | 27  |
| 3    | The principal-type algorithm  |  | 30  |
|      | 3A                            | Principal types and their history            | 31  |
|      | 3 <b>B</b>                    | Type-substitutions                           | 34  |
|      | 3C                            | Motivating the PT algorithm                  | 38  |
|      | 3D                            | Unification                                  | 40  |
|      | 3E                            | The PT algorithm                             | 44  |
| 4    | Type assignment with equality |  | 52  |
|      | 4A                            | The equality rule                            | 52  |
|      | 4B                            | Semantics and completeness                   | 57  |
| 5    | A ve                          | rsion using typed terms                      | 63  |
|      | 5A                            | Typed terms                                  | 63  |
|      | 5B                            | Reducing typed terms                         | 67  |
|      | 5C                            | Normalization theorems                       | 71  |
| 6    | The                           | correspondence with implication              | 74  |
|      | 6A                            | Intuitionist implicational logic             | 74  |
|      | 6B                            | The Curry-Howard isomorphism                 | 79  |
|      | 6C                            | Some weaker logics                           | 85  |
|      | 6D                            | Axiom-based versions                         | 88  |
| 7    | The                           | converse principal-type algorithm            | 93  |
|      | 7A                            | The converse PT theorems                     | 93  |
|      | 7 <b>B</b>                    | Identifications                              | 95  |

| Contents |
|----------|
|----------|

|        | 7C                | The converse PT proof                              | 96  |
|--------|-------------------|--|-----|
|        | 7D                | Condensed detachment                               | 102 |
| 8      | Coun              | 108  |     |
|        | 8A                | Inhabitants  | 108 |
|        | 8B                | Examples of the search strategy                    | 114 |
|        | 8C                | The search algorithm                               | 118 |
|        | 8D                | The Counting algorithm                             | 124 |
|        | 8E                | The structure of a nf-scheme                       | 127 |
|        | 8F                | Stretching, shrinking and completeness             | 132 |
| 9      | Technical details |  | 140 |
|        | 9A                | The structure of a term                            | 140 |
|        | 9B                | Residuals  | 144 |
|        | 9C                | The structure of a $TA_{\lambda}$ -deduction       | 148 |
|        | 9D                | The structure of a type                            | 151 |
|        | 9E                | The condensed structure of a type                  | 153 |
|        | 9F                | Imitating combinatory logic in $\lambda$ -calculus | 157 |
| Answ   | 161               |  |     |
| Biblic | 169               |  |     |
| Table  | 177               |  |     |
| Index  | 179               |  |     |

viii