

TABLE OF CONTENTS

	PAGE
PREFACE	V
EXPLANATION OF NOTATIONS.	XIII
INTRODUCTION	1
A. The Analysis of Substitution	1
B. The Russell Paradox	4
C. Plan of the Work	5
D. Historical Sketch	8
CHAPTER 1. FORMAL SYSTEMS	12
A. The Approach to Formal Systems.	12
B. Definition of a Formal System	14
C. Philosophy of Formal Systems	19
D. Linguistic Aspects of a Formal System	23
E. Special Forms of Formal Systems	28
S. Supplementary Topics	33
CHAPTER 2. EPITHEORY	40
A. The Nature of Epittheory.	40
B. Techniques of Induction	44
C. Variables	52
D. Replacement and Monotone Relations	57
E. Theory of Definition.	62
S. Supplementary Topics	76
CHAPTER 3. LAMBDA-CONVERSION	80
A. Variables and Functions in Mathematics	80
B. Functional Abstraction	83
C. Morphology of a Formal λ -Applicative System	87
D. Theoretical Rules of the Calculuses of Lambda-Conversion	89
E. Substitution Prefixes	94
S. Supplementary Topics	104
CHAPTER 4. THE CHURCH-ROSSER THEOREM	108
A. General Formulation	108
B. Property (D)	115

	PAGE
C. Property (E)	123
D. Extension to Include η -Conversion	131
E. The Second Church-Rosser Theorem.	139
F. Theorems on Order	144
S. Supplementary Topics.	149
CHAPTER 5. INTUITIVE THEORY OF COMBINATORS.	151
A. Preliminary Discussion	152
B. Interdefinability of Simple Combinators	155
C. Terminology and Notation	160
D. Properties of B	163
E. Combinators Related to S	168
F. Theorems on Order of Combinators	175
G. Paradoxical Combinators	177
H. Definitional Independence of Combinators	179
S. Supplementary Topics.	184
CHAPTER 6. SYNTHETIC THEORY OF COMBINATORS	186
A. Analysis of Combinatorial Completeness	186
B. Basic Properties of \mathcal{H}	194
C. The Combinatory Axioms	197
D. Theory of the Substitution Prefix	205
E. Equivalence of \mathcal{H} and Lambda-Conversion.	209
F. Theory of Strong Reduction	218
S. Supplementary Topics.	236
CHAPTER 7. LOGISTIC FOUNDATIONS	239
A. Preliminaries	239
B. The Combinatory Rules	241
C. The Systems \mathcal{Q}	245
D. The Systems \mathcal{H}	248
E. Modifications and Generalizations.	250
S. Supplementary Topics.	255
CHAPTER 8. INTRODUCTION TO ILLATIVE COMBINATORY LOGIC :	257
A. The Russell Paradox	258
B. Alternative Explanations of the Paradox.	260
C. The Notion of Functionality	262
D. Relations to Other Illative Concepts.	266
E. Formal Preliminaries	268
S. Supplementary Topics.	273

	PAGE
CHAPTER 9. THE BASIC THEORY OF FUNCTIONALITY	277
A. Preliminaries	277
B. The Subject-Construction Theorem	282
C. The Subject-Conversion Theorems.	293
D. The Stratification Theorem and Its Consequences	304
E. Analogies with Propositional Algebra	312
F. Inferential Forms of Functionality	315
S. Supplementary Topics	344
CHAPTER 10. THE STRONGER THEORIES OF FUNCTIONALITY	345
A. Preliminaries	345
B. General Properties of Deductions	353
C. FR-Deductions	360
D. The Restricted Theories	365
E. Finite Formulation	368
APPENDIX A. LIST OF BASIC CONSTANTS.	377
APPENDIX B. LIST OF PROPERTIES OF RELATIONS	380
BIBLIOGRAPHY	382
INDEX	395