

# CONTENTS

- 1 INTRODUCTION 3
  - 1. *What logic is about* 4
  - 2. *Soundness and truth* 5
  - 3. *Soundness and necessary truth* 7
  - 4. *Parenthetical remarks* 9
  - 5. *Logical form* 13
  - 6. *Artificial languages* 16
- 2 FURTHER PRELIMINARIES 20
  - 1. *Use and mention* 21
  - 2. *Sense and denotation* 22
  - 3. *Variables* 24
  - 4. *Sentence-forms* 27
  - 5. *Sets* 33
  - 6. *Object-language and metalanguage* 40
- 3 THE FORMALIZED LANGUAGE  $\mathcal{L}$  44
  - 1. *The grammar of  $\mathcal{L}$*  44
  - 2. *Explanations and examples* 45
  - 3. *Additional syntactic terminology* 49
  - 4. *Notational conventions* 51
- 4 INTERPRETATIONS AND VALIDITY 54
  - 1. *Interpretations of  $\mathcal{L}$*  54
  - 2. *Truth* 57
  - 3. *Validity, consequence, consistency* 63
- 5 TRANSLATING THE NATURAL LANGUAGE INTO  $\mathcal{L}$  69
  - 1. *Introduction* 70
  - 2. *Interpretation and translation* 75
  - 3. *Translating connectives and quantifiers* 78

6 TAUTOLOGOUS SENTENCES 87

1. *Introduction* 87
2. *Definition of 'tautology'* 89
3. *Tautologous SC sentences; truth-tables* 90
4. *Deciding whether sentences are tautologous* 94
5. *Further properties* 95
6. *Rules of derivation for SC sentences* 96

7 RULES OF INFERENCE FOR  $\mathcal{L}$  111

1. *Basic rules; derivability* 111
2. *Examples and explanations* 113
3. *The short-cut rules EG, ES, and Q* 120
4. *Strategy* 126
5. *Theorems of logic* 127

8 SOME METATHEOREMS 133

1. *Replacement, negation and duality, prenex normal form* 133
2. *Soundness and consistency* 139
3. *Completeness* 142
4. *A proof procedure for valid sentences* 147

9 IDENTITY AND TERMS 151

1. *Identity; the language  $\mathcal{L}_1$*  151
2. *Parenthetical remarks* 156
3. *Terms; the language  $\mathcal{L}'$*  158

10 AXIOMS FOR  $\mathcal{L}_1$  164

1. *Introduction* 164
2. *Sentential calculus* 171
3. *Quantification theory* 175
4. *Identity, further quantification theory, and substitution* 177
5. *Proofs from assumptions* 180

11 FORMALIZED THEORIES 183

1. *Introduction* 183
2. *Aristotelian syllogistic* 188
3. *The theory of betweenness* 191
4. *Groups; Boolean algebras* 193
5. *Definitions* 197

12 A BRIEF OUTLINE OF THE HISTORY OF LOGIC 205

1. *Ancient logic* 206
2. *Medieval logic* 217
3. *Modern logic* 223

BIBLIOGRAPHY 231

INDEX 233