## CONTENTS

Preface	vii	
Chapter 1. Real and Complex Numbers. Elementary Algebra	1	
1.1. Introduction. The Real-number System1.2. Powers, Roots, Logarithms, and Factorials. Sum and Product Notation	2 4	
1.3. Complex Numbers	7	
1.4. Miscellaneous Formulas         . <th .<<="" td=""><td>10 12</td></th>	<td>10 12</td>	10 12
1.6. Algebraic Equations: General Theorems	12	
1.7. Factoring of Polynomials and Quotients of Polynomials. Partial Frac-		
tions	19	
1.8. Linear, Quadratic, Cubic, and Quartic Equations	22	
1.9. Systems of Simultaneous Equations	24	
1.10. Related Topics, References, and Bibliography	27	
Chapter 2. Plane Analytic Geometry	28	
2.1. Introduction and Basic Concepts.	<b>29</b>	
2.2. The Straight Line	35	
2.3. Relations Involving Points and Straight Lines	37	
2.4. Second-order Curves (Conic Sections)	39	
2.5. Properties of Circles, Ellipses, Hyperbolas, and Parabolas.	46	
2.6. Higher Plane Curves	51	
2.7. Related Topics, References, and Bibliography	<b>5</b> 2	
Chapter 3. Solid Analytic Geometry	54	
3.1. Introduction and Basic Concepts.	55	
3.2. The Plane	64	
3.3. The Straight Line	66	
3.4. Relations Involving Points, Planes, and Straight Lines	67	
3.5. Quadric Surfaces.	<b>71</b>	
3.6. Related Topics, References, and Bibliography	79	
Chapter 4. Functions and Limits. Differential and Integral Calculus	<b>8</b> 0	
4.1. Introduction	82	
4.2. Functions.	82	
4.3. Point Sets, Intervals, and Regions	84	
4.4. Limits, Continuous Functions, and Related Topics	87	
4.5. Differential Calculus	92	
4.6. Integrals and Integration	99	
	ix	

4.7.	Mean-value Theorems. Values of Indeterminate Forms. Weier		
	Approximation Theorems		115
	Infinite Series, Infinite Products, and Continued Fractions		118
4.9.	Tests for the Convergence and Uniform Convergence of Infinite Ser	ies and	
	Improper Integrals		124
4.10.	Representation of Functions by Infinite Series and Integrals.	Power	
	Series and Taylor's Expansion		128
4.11.			131
	Related Topics, References, and Bibliography		140
			_
Chap	ter 5. Vector Analysis	• •	141
5.1.	Introduction		142
	Vector Algebra		143
53		• •	147
	Scalar and Vector Fields		149
	Differential Operators		149
			153
0.0. F 7	Integral Theorems .	• •	
	Specification of a Vector Field in Terms of Its Curl and Divergence		160
5.8.	Related Topics, References, and Bibliography	• •	162
Chan	ter 6. Curvilinear Coordinate Systems		164
-			
	Introduction	• •	164
	Curvilinear Coordinate Systems		165
	Representation of Vectors in Terms of Components		167
6.4.	Orthogonal Coordinate Systems. Vector Relations in Terms of C	)rthog-	
	onal Components		169
6.5.	Formulas Relating to Special Orthogonal Coordinate Systems .	• •	181
6.6.	Related Topics, References, and Bibliography		182
	ter 7. Functions of a Complex Variable		183
7.1.	Introduction		184
	Functions of a Complex Variable. Regions of the Complex-n		
	Plane.		184
73	Analytic (Regular, Holomorphic) Functions.	•••	188
	Treatment of Multiple-valued Functions.		189
1.T. 7 E	Integral Theorems and Series Expansions	• •	191
7.0.	Represent Linebrend Schemberking	• •	191
7.0.	Zeros and Isolated Singularities	· ·	
	Residues and Contour Integration.		197
7.8.	Analytic Continuation	• •	199
7.9.	Conformal Mapping.	• •	200
	Functions Mapping Specified Regions onto the Unit Circle		213
7.11.	Related Topics, References, and Bibliography		213
Chart	ter 8. The Laplace Transformation and Other Integral		
	ransformations.		215
	Introduction		216
8.2.	The Laplace Transformation		216
8.3.	Correspondence between Operations on Object and Result Function	ons .	219
8.4.	Tables of Laplace-transform Pairs and Computation of Inverse L	aplace	
	Transforms		222
8.5.	"Formal" Laplace Transformation of Impulse-function Terms		227

ж

	Some Other Functional Transformations	$227 \\ 231$
	ster 9. Ordinary Differential Equations	233
-		004
	Introduction	234
	First-order Equations	237
	Linear Differential Equations	242
	Linear Differential Equations with Constant Coefficients	253
	Nonlinear Second-order Equations	264
9.6.	Pfaffian Differential Equations	270
9.7.	Related Topics, References, and Bibliography	272
Chap	ter 10. Partial Differential Equations	273
10.1.	Introduction and Survey	<b>274</b>
	Partial Differential Equations of the First Order	<b>276</b>
10.3.	Hyperbolic, Parabolic, and Elliptic Partial Differential Equations.	
	Characteristics	288
10.4.	Linear Partial Differential Equations of Physics. Particular Solutions.	297
10.5.	Integral-transform Methods	310
10.6.	Related Topics, References, and Bibliography	314
Chap	ter 11. Maxima and Minima	315
11.1.	Introduction	316
	Maxima and Minima (Extreme Values) of Functions of One Real Variable	316
	Maxima and Minima (Extreme Values) of Functions of Two or More Real	010
11.0.	Variables.	317
114	Calculus of Variations. Maxima and Minima of Definite Integrals	320
	Solution of Variation Problems in Terms of Differential Equations	322
	Solution of Variation Problems by Direct Methods.	328
	Related Topics, References, and Bibliography	329
		020
	ter 12. Definition of Mathematical Models: Modern (Abstract)	
Α	lgebra and Abstract Spaces	331
12 1	Introduction	332
	Algebra of Models with a Single Defining Operation: Groups	336
	Algebra of Models with Two Defining Operation: Rings, Fields, and Inte-	000
12.0.	gral Domains.	340
19 4	Models Involving More Than One Class of Mathematical Objects: Linear	010
12.7.	Vector Spaces and Linear Algebras	342
19 5	Models Permitting the Definition of Limiting Processes: Topological	342
12.0.		344
10.6	Spaces.	
	Order.	347
12.7.	Combination of Models: Direct Products, Product Spaces, and Direct	0.40
	Sums.	348
	Boolean Algebras	350
12.9.	Related Topics, References, and Bibliography	353
Chap	ter 13. Matrices. Quadratic and Hermitian Forms	355
13.1.	Introduction.	356
	Matrix Algebra and Matrix Calculus.	356
	Matrices with Special Symmetry Properties.	362

CONTENTS

xi

CONTENTS	xii
13.4. Equivalent Matrices. Eigenvalues, Diagonalization, and Related Topics 13.5. Quadratic and Hermitian Forms	364 368
13.6. Related Topics, References, and Bibliography	372
Chapter 14. Linear Vector Spaces and Linear Transformations (Linear Operators). Representation of Mathematical Models in Terms of	974
Matrices	374
14.1. Introduction. Reference Systems and Coordinate Transformations.	376
14.2. Linear Vector Spaces	378
14.3. Linear Transformations (Linear Operators)	382
14.4. Linear Transformations of a Normed or Unitary Vector Space into Itself.	n0 4
Hermitian and Unitary Transformations (Operators)	384
14.5. Matrix Representation of Vectors and Linear Transformations (Opera-	388
tors)         . <td>390</td>	390
14.6. Change of Reference System         14.7. Representation of Inner Products.         Orthonormal Bases	392
14.8. Eigenvectors and Eigenvalues of Linear Operators	396
14.9. Group Representations and Related Topics	406
14.10. Mathematical Description of Rotations	411
14.11. Related Topics, References, and Bibliography	418
11.11. Infance repres, increases, and Distrigraphy	110
Chapter 15. Linear Integral Equations, Boundary-value Problems, and Eigenvalue Problems	420
15.1. Introduction	422
15.2. Functions as Vectors. Expansions in Terms of Orthogonal Functions	423
15.3. Linear Integral Transformations and Linear Integral Equations	428
15.4. Linear Boundary-value Problems and Eigenvalue Problems Involving	120
Differential Equations.	438
15.5. Green's Functions. Relation of Boundary-value Problems and Eigen-	100
value Problems to Integral Equations	451
15.6. Potential Theory	456
15.7. Related Topics, References, and Bibliography	467
Chapter 16. Representation of Mathematical Models: Tensor Algebra	
and Analysis	469
16.1. Introduction	470
16.2. Absolute and Relative Tensors	473
16.3. Tensor Algebra: Definition of Basic Operations.	476
16.4. Tensor Algebra: Invariance of Tensor Equations	479
16.5. Symmetric and Skew-symmetric Tensors	479
16.6. Local Systems of Base Vectors	481
16.7. Tensors Defined on Riemann Spaces. Associated Tensors	482
16.8. Scalar Products and Related Topics	485
16.9. Tensors of Rank Two (Dyadics) Defined on Riemann Spaces	487
16.10. The Absolute Differential Calculus. Covariant Differentiation	488
16.11. Related Topics, References, and Bibliography	496
Chapter 17. Differential Geometry	497
17.1. Curves in the Euclidean Plane	498
17.2. Curves in Three-dimensional Euclidean Space	501
17.3. Surfaces in Three-dimensional Euclidean Space.	505

.

CON	TEN	TS
-----	-----	----

17.4.	Curved Spaces	$515 \\ 520$
Chap	ter 18. Probability Theory and Random Processes	521
	Introduction	523
18.2.	Definition and Representation of Probability Models	524
	One-dimensional Probability Distributions	529
18.4.	Multidimensional Probability Distributions	538
18.5.	Functions of Random Variables. Change of Variables	550
18.6.	Convergence in Probability and Limit Theorems	554
	Special Techniques for Solving Probability Problems	557
18.8.	Special Probability Distributions	559
18.9.	Description of Random Processes .	571
18.10.	Generalized Fourier Analysis for Stationary Random Processes. Corre-	
	lation Functions and Spectra	577
18.11.	Examples of Random Processes	583
18.12.	Related Topics, References, and Bibliography	585
Chan	ter 19. Mathematical Statistics	587
•		
	Introduction to Statistical Methods	588
19.2.	Statistical Description. Definition and Computation of Random-sample	
	Statistics.	591
19.3.	General-purpose Probability Distributions	597
19.4.	Estimation of Parameters	599
19.5.	Sampling Distributions.	603
	Tests of Statistical Hypotheses	<b>6</b> 09
19.7.	Some Statistics, Sampling Distributions, and Tests for Multivariate Dis-	
	tributions	619
19.8.	Related Topics, References, and Bibliography	626
Chapt	ter 20. Numerical Calculations and Finite Differences	627
	Introduction	629
20.2.	Numerical Solution of Equations	630
20.3.	Linear Simultaneous Equations and Matrix Inversion. Eigenvalues and	
	Eigenvectors of Matrices	635
20.4.	Finite Differences and Difference Equations	641
20.5.	Polynomial Interpolation, Numerical Harmonic Analysis, and Other	
	Approximation Methods	650
20.6.	Numerical Differentiation and Integration	662
	Numerical Solution of Ordinary Differential Equations	666
20.8.	Numerical Solution of Partial Differential Equations, Boundary-value	
	Problems, and Integral Equations	671
20.9.	Related Topics, References, and Bibliography	680
-	ter 21. Special Functions	682
	Introduction	684
	The Elementary Transcendental Functions.	684
	Some Functions Defined by Transcendental Integrals	695
	The Gamma Function and Related Functions	697
21.5.	Binomial Coefficients and Factorial Polynomials. Bernoulli Polynomials	
	and Bernoulli Numbers.	700

		C	ONT	'EN'	гs										xiv
<ul> <li>21.6. Elliptic Functions, Elliptic Integrals, and Related Functions.</li> <li>21.7. Orthogonal Polynomials</li> <li>21.8. Cylinder Functions, Associated Legendre Functions, and Spherical Har-</li> </ul>													703 722		
monics												·		•	727
21.9. Step Fun	ctions and Sy	mbolic	Imp	ulse	Fun	ctio	$\mathbf{ns}$	•	·	٠	·				740
21.10. Reference	es and Bibliog	raphy		•	·	•	•	•	•	·	•	·	•	·	746
Appendix A.	Formulas De	escribi	ng P	lane	Fig	gure	<b>s a</b> :	nd	Sol	lide	<b>3</b> .				747
Appendix B.	Plane and Sj	pheric	al Tr	igor	om	etry	y		•						751
Appendix C.	Permutation	is, Cor	nbin	atio	ns,	and	Re	late	ed '	Гој	pics	۱.	•		760
Appendix D.	Tables of <b>H</b>	ourie	r Ex	pans	sion	s a	nd	La	pla	ıce-	-tra	insi	for	m	
Pairs .		· ·	· ·	•	·	·	•	•	·	·	·	·	·	·	763
Appendix E.	Tables of Inc	lefinit	e an	d De	fini	te I	nte	gra	ls		•				787
Appendix F.	Numerical T	ables							•		•			•	827
Squares.												•	•		828
Logarithms.												•		•	831
Trigonometri	Functions.				•			•	•	•	•	•	·	•	848
Exponential a	nd Hyperboli	c Func	tions					•	•	•	•	•	•	•	856
Natural Loga	rithms				•	•	•	•	•	•	•	·	•	·	863
											•	•	•		865
	al .										·	·	•	•	866
Exponential a	and Related In	ntegral	s		•		•						·		867
Complete Elli	iptic Integrals			•	•	٠	•	·	·	·	٠	·	·	·	871
	d Their Recip														872
Binomial Coe	fficients	• •		•	•								•		872
Gamma and I	Factorial Fun	etions		·	·	•							·		873
Bessel Functi	ons		· ·	•	•	·	·	·	•	·	·	·	·	•	875
Legendre Pol	ynomials .	· ·		·	•	·	•	•	·	·	·	·	·	•	898 899
	on										·	·	·	•	- 899 - 900
	bution Areas										·	·	·	·	900
	ordinates.			•			•								901
Distribution	of t	· ·	• •	·	٠	·	•	•	·	·	·	•	·	•	902 903
	of $\chi^2$												•	•	903 904
Distribution	of <i>F</i> .	• •	• •	•	·	·	·	•	•	·	•	·	•	•	504
Glossary of Sym	ibols and Note	ations	•		•	•	•	•	•	•	•	•	•	•	909
Index					•	•	•	•	•	•	•	•	•	•	915