Contents

PREFACE xv

TO THE TEACHER xvii

TO THE STUDENT xxi

POSSIBLE WORLDS 1

THIS AND OTHER POSSIBLE WORLDS 1 1. The realm of possibilities 1 What are the limits to the possible? 2 Possibility is not the same as conceivability 3 Possible worlds: actual and non-actual Logical possibility distinguished from other kinds 6 The constituents of possible worlds 7 PROPOSITIONS, TRUTH, AND FALSITY 2. 9 Truth and falsity defined 9 Truth in a possible world 11 Truth in the actual world 12 The myth of degrees of truth 12 3. **PROPERTIES OF PROPOSITIONS** 13 Possibly true propositions 13 Possibly false propositions 13 14 Contingent propositions Contradictories of propositions 14 Noncontingent propositions 15 Necessarily true propositions 16 Necessarily false propositions 17 More about contradictory propositions 18 Some main kinds of noncontingent propositions 19 Summary 24 Symbolization 25

4. RELATIONS BETWEEN PROPOSITIONS 28 Inconsistency 28 Consistency 30 Implication 31 Equivalence 35 Symbolization 41

5. SETS OF PROPOSITIONS 42 Truth-values of proposition-sets 42 Modal properties of proposition-sets 42 Modal relations between proposition-sets 44 Minding our "P's and "Q"s 47

6. MODAL PROPERTIES AND RELATIONS PICTURED ON WORLDS-DIAGRAMS 48

Worlds-diagrams for modal properties 49 Worlds-diagrams for modal relations 50 Interpretation of worlds-diagrams 50 A note on history and nomenclature 53 Capsule descriptions of modal relations 54 Appendix to section 6 57

7. IS A SINGLE THEORY OF TRUTH ADEQUATE FOR BOTH CONTINGENT AND NONCONTINGENT PROPOSITIONS? 58

8. THE "POSSIBLE WORLDS" IDIOM 62

2 PROPOSITIONS 65

1. INTRODUCTION 65

2. THE BEARERS OF TRUTH-VALUES 65

Thesis 1: Such things as beliefs, statements, assertions, remarks, hypotheses, and theories are the bearers of truth and falsity. 68

Thesis 2: Acts of believing (stating, asserting, etc.) are the bearers of truth-values. 68

Thesis 3: That which is believed, stated, etc., is what is true or false. 71

- Thesis 4: Sentences are the bearers of truth-values. 71
- Thesis 5: Sentence-tokens are the bearers of truth-values. 73
- Thesis 6: Sentence-types are the bearers of truth-values. 74

Thesis 7: Context-free sentences are the bearers of truth-values. 75

- Thesis 8: Context-free sentence-tokens are those things to which truth and falsity may be attributed. 76
- Thesis 9: Context-free sentence-types are those things to which truth and falsity may be attributed. 76

Thesis 10: Propositions are those things to which truth and falsity may be attributed. 79

Thesis 11: Propositions are to be identified with the meanings of sentences. 80

Thesis 12: Propositions are to be identified with sets of possible worlds. 82

Thesis 13: Propositions are abstract entities in their own right; that is, they are sui generis, they are not to be identified with any other kind of abstract entity. 84 Categorial differences between sentences and propositions 86 One final note 86

3. THE STRUCTURE OF PROPOSITIONS: A SPECULATIVE THEORY 87

Concepts 87 Attributes of concepts 90 Identity conditions for concepts 92 Analysis of propositions 94 Identity conditions for propositions 96

4. ON REFERRING TO SENTENCES AND TO PROPOSITIONS 97

Techniques for referring to sentences 97 Basic techniques for referring to propositions 98 Advanced technique for referring to propositions: contextfree references 100 Untensed verbs in context-free references 103

5. THE OMNITEMPORALITY OF TRUTH 104

6. PROPOSITIONS, SENTENCES, AND POSSIBLE WORLDS 108 The uni-linguo proviso 110 The linguo-centric proviso 111 Securing reference to propositions 111

7. SENTENTIAL AMBIGUITY AND POSSIBLE-WORLDS TESTING 113

Sentential ambiguity 113 The method of possible-worlds testing 114 Janus-faced sentences 119

8. POSSIBLE-WORLDS PARABLES 121

Case Study 1: The thesis that persons (creatures) who lack a language cannot have reflective beliefs 122 Case Study 2: The thesis that persons (creatures) who lack a language cannot believe necessary truths 125 Case Study 3: The thesis that a justified belief in a true proposition constitutes knowledge 126

5 KNOWLEDGE 129

- 1. THE SUBJECT MATTER AND THE SCIENCE OF LOGIC 129
- 2. THE NATURE OF KNOWLEDGE 130

- 1. Is it a necessary condition of the truth of a's knowing that P, that P should be true? 131
- 2. Is it a necessary condition of a's knowing that P, that a believe that P? 133
- 3. Is it a necessary condition of a's knowing that P, that a be justified in believing that P? 136
- 4. What might the missing fourth necessary condition for a's knowing that P be? 137
- 3. THE LIMITS OF HUMAN KNOWLEDGE 139 The known and the unknown 139 The knowable and the unknowable 140

EXPERIENTIAL AND RATIOCINATIVE KNOWLEDGE 142 Experiential knowledge 142 Ratiocinative knowledge 144 Appendix to section 4 149

5. EMPIRICAL AND A PRIORI KNOWLEDGE 149

Definitions of "empirical" and "a priori" 150 The non-exhaustiveness and non-exclusiveness of the experiential/ratiocinative distinction 151 The exhaustiveness and exclusiveness of the empirical/ a priori distinction 152 Is a priori knowledge certain? 155

6. EPISTEMIC AND MODAL STATUS CONSIDERED TOGETHER 156

- 1. Are there any contingent propositions which are knowable empirically? 157
- 2. Are there any contingent propositions which are knowable both experientially and ratiocinatively? 158
- 3. Are there any contingent propositions which are knowable ratiocinatively but which are not knowable experientially? 163
- 4. Are there any contingent propositions which are knowable by other than experiential or ratiocinative means? 164
- 5. Are there any contingent propositions which are unknowable? 167
- 6. Are there any noncontingent propositions knowable empirically? 168
- 7. Are there any noncontingent propositions which are knowable both experientially and ratiocinatively? 170
- 8. Are there any noncontingent propositions which are knowable ratiocinatively but which are not knowable experientially? 170

Appendix to section 6: a complete classificatory scheme for the epistemic and modal distinctions 174

THE SCIENCE OF LOGIC: AN OVERVIEW 179

1. INTRODUCTION 179

2. THE METHOD OF ANALYSIS 180

The objects of philosophical analysis 180 Three levels of analysis 181 The idea of a complete analysis 183 The need for a further kind of analysis 184 Possible-worlds analysis 185 Degrees of analytical knowledge 187

3. THE PARADOX OF ANALYSIS 189 Moore's problem 189 A Moorean solution 190

4. THE METHOD OF INFERENCE 192

The nature of inference 193 Valid and invalid propositional inferences 195 Determining the validity of inferences: the problem of justification 196 Rules of inference 198 What kind of rule is a rule of inference? 200 Inference and the expansion of knowledge 201

5. INFERENCE WITHIN THE SCIENCE OF LOGIC 205 Inference within axiomatic systems: the example of S5 205 Inference within natural deduction systems 210 The theoretical warrant of the method of direct proof 215

6. A PHILOSOPHICAL PERSPECTIVE ON LOGIC AS A WHOLE 218

The indispensability of modal concepts within propositional logics 218 Problems about the reduction principles 220 Problems about the paradoxes 224 Relevance logics 228 The move to predicate logic 230 Traditional syllogistic 232 Modern predicate logic 233 Modal notions in predicate logic 236 Modalities de dicto and de re 237 Heterogeneous and homogeneous possible worlds 239 Is there really a logic of concepts? 240

5 TRUTH-FUNCTIONAL PROPOSITIONAL LOGIC 247

1. INTRODUCTION 247

2. TRUTH-FUNCTIONAL OPERATORS 247

The uses of "not" and "it is not the case that" 249 The uses of "and" 252 The uses of "or" 257 Interlude: compound sentences containing two or more sentential operators 261 The uses of "if ... then ..." 263 The uses of "if and only if" 269 Appendix: truth-tables for wffs containing three or more letters 272

- 3. EVALUATING COMPOUND SENTENCES 273 A note on two senses of "determined" 277
- ELEMENTARY TRUTH-TABLE TECHNIQUES FOR REVEALING MODAL STATUS AND MODAL RELATIONS 279 Modal status 279 Modal relations 284 Deductive validity 290
- 5. ADVANCED TRUTH-TABLE TECHNIQUES 294 Corrected truth-tables 294 Reduced truth-tables 297
- 6. THE CONCEPT OF FORM 301 Sentences and sentential forms in a logic 301 The relationship between sentences and sentence-forms 302
- 7. EVALUATING SENTENCE-FORMS 306 The validity of sentence-forms 306 Modal relations 308 Implication 308 Equivalence 309 Inconsistency 309 Argument-forms and deductive validity 310
- 8. FORM IN A NATURAL LANGUAGE 311

9. WORLDS-DIAGRAMS AS A DECISION PROCEDURE FOR TRUTH-FUNCTIONAL PROPOSITIONAL LOGIC 313

10. A SHORTCUT FORMAL METHOD: REDUCTIO AD ABSURDUM TESTS 315 Summary 320

D MODAL PROPOSITIONAL LOGIC 323

1. INTRODUCTION 323

2

- MODAL OPERATORS 323 Non-truth-functionality 323 Modal and nonmodal propositions; modalized and nonmodalized formulae 324 The interdefinability of the monadic and dyadic modal operators 327
- 3. SOME PROBLEMATIC USES OF MODAL EXPRESSIONS 329 "It is possible that" 329 Problems with the use of "it is necessary that"; the modal fallacy; absolute and relative necessity 330
- 4. THE MODAL STATUS OF MODAL PROPOSITIONS 333
- 5. THE OPERATOR "IT IS CONTINGENTLY TRUE THAT" 337
- 6. ESSENTIAL PROPERTIES OF RELATIONS 339
- TWO CASE STUDIES IN MODAL RELATIONS: a lighthearted interlude 345

Case study 1: the pragmatics of telling the truth 345 Case study 2: an invalid inference and an unwitting impossible description 347

8. USING WORLDS-DIAGRAMS TO ASCERTAIN THE VALIDITY OF MODALIZED FORMULAE 350 Applications 351 The validity of the axioms of S5 356 The nonvalidity of the axiom set for S6 358

- 9. A SHORTCUT FORMAL METHOD FOR DETERMINING THE VALIDITY OF MODALIZED FORMULAE: modal reductios 359
- 10. THE NUMBER OF FORMALLY NON-EQUIVALENT SENTENCE-FORMS CONSTRUCTIBLE ON N SENTENCE-VARIABLES 365

11. LOOKING BEYOND MODAL LOGIC TO INDUCTIVE LOGIC 370

The cardinality of a class and other concepts of class size 371 The concept of contingent content 372 Monadic modal functors 375 What are the prospects for a fully-developed inductive logic? 379 The concept of probabilification 381 A dyadic modal functor for the concept of probabilification 382

INDEX 385

xiv