

# CONTENTS

PREFACE	v
HINTS TO THE READER	xi
CHAPTER I. INDUCTION	3
1. Experience and belief. 2. Suggestive contacts. 3. Supporting contacts. 4. The inductive attitude	
Examples and Comments on Chapter I, 1-14. [12. Yes and No. 13. Experience and behavior. 14. The logician, the mathematician, the physicist, and the engineer.]	
CHAPTER II. GENERALIZATION, SPECIALIZATION, ANALOGY	12
1. Generalization, specialization, analogy, and induction. 2. Generalization. 3. Specialization. 4. Analogy. 5. Generalization, specialization, and analogy. 6. Discovery by analogy. 7. Analogy and induction	
Examples and Comments on Chapter II, 1-46; [First Part, 1-20; Second Part, 21-46]. [1. The right generalization. 5. An extreme special case. 7. A leading special case. 10. A representative special case. 11. An analogous case. 18. Great analogies. 19. Clarified analogies. 20. Quotations. 21. The conjecture <i>E</i> . 44. An objection and a first approach to a proof. 45. A second approach to a proof. 46. Dangers of analogy.]	
CHAPTER III. INDUCTION IN SOLID GEOMETRY	35
1. Polyhedra. 2. First supporting contacts. 3. More supporting contacts. 4. A severe test. 5. Verifications and verifications. 6. A very different case. 7. Analogy. 8. The partition of space. 9. Modifying the problem. 10. Generalization, specialization, analogy. 11. An analogous problem. 12. An array of analogous problems. 13. Many problems may be easier than just one. 14. A conjecture. 15. Prediction and verification. 16. Again and better. 17. Induction suggests deduction, the particular case suggests the general proof. 18. More conjectures	
Examples and Comments on Chapter III, 1-41. [21. Induction: adaptation of the mind, adaptation of the language. 31. Descartes' work on polyhedra. 36. Supplementary solid angles, supplementary spherical polygons.]	

## CHAPTER IV. INDUCTION IN THE THEORY OF NUMBERS 59

1. Right triangles in integers. 2. Sums of squares. 3. On the sum of four odd squares. 4. Examining an example. 5. Tabulating the observations. 6. What is the rule? 7. On the nature of inductive discovery. 8. On the nature of inductive evidence

Examples and Comments on Chapter IV, 1-26. [1. Notation. 26. Dangers of induction.]

## CHAPTER V. MISCELLANEOUS EXAMPLES OF INDUCTION 76

1. Expansions. 2. Approximations. 3. Limits. 4. Trying to disprove it. 5. Trying to prove it. 6. The role of the inductive phase

Examples and Comments on Chapter V, 1-18. [15. Explain the observed regularities. 16. Classify the observed facts. 18. What is the difference?]

## CHAPTER VI. A MORE GENERAL STATEMENT 90

1. Euler. 2. Euler's memoir. 3. Transition to a more general viewpoint. 4. Schematic outline of Euler's memoir

Examples and Comments on Chapter VI, 1-25. [1. Generating functions. 7. A combinatorial problem in plane geometry. 10. Sums of squares. 19. Another recursion formula. 20. Another Most Extraordinary Law of the Numbers concerning the Sum of their Divisors. 24. How Euler missed a discovery. 25. A generalization of Euler's theorem on  $\sigma(n)$ .]

## CHAPTER VII. MATHEMATICAL INDUCTION 108

1. The inductive phase. 2. The demonstrative phase. 3. Examining transitions. 4. The technique of mathematical induction

Examples and Comments on Chapter VII, 1-18. [12. To prove more may be less trouble. 14. Balance your theorem. 15. Outlook. 17. Are any  $n$  numbers equal?]

## CHAPTER VIII. MAXIMA AND MINIMA 121

1. Patterns. 2. Example. 3. The pattern of the tangent level line. 4. Examples. 5. The pattern of partial variation. 6. The theorem of the arithmetic and geometric means and its first consequences

Examples and Comments on Chapter VIII, 1-63; [First Part, 1-32; Second Part, 33-63]. [1. Minimum and maximum distances in plane geometry. 2. Minimum and maximum distances

in solid geometry. 3. Level lines in a plane. 4. Level surfaces in space. 11. The principle of the crossing level line. 22. The principle of partial variation. 23. Existence of the extremum. 24. A modification of the pattern of partial variation: An infinite process. 25. Another modification of the pattern of partial variation: A finite process. 26. Graphic comparison. 33. Polygons and polyhedra. Area and perimeter. Volume and surface. 34. Right prism with square base. 35. Right cylinder. 36. General right prism. 37. Right double pyramid with square base. 38. Right double cone. 39. General right double pyramid. 43. Applying geometry to algebra. 45. Applying algebra to geometry. 51. Right pyramid with square base. 52. Right cone. 53. General right pyramid. 55. The box with the lid off. 56. The trough. 57. A fragment. 62. A post office problem. 63. A problem of Kepler.]

## CHAPTER IX. PHYSICAL MATHEMATICS

142

1. Optical interpretation. 2. Mechanical interpretation. 3. Re-interpretation. 4. Jean Bernoulli's discovery of the brachistochrone. 5. Archimedes' discovery of the integral calculus

Examples and Comments on Chapter IX, 1-38. [3. Triangle with minimum perimeter inscribed in a given triangle. 9. Traffic center of four points in space. 10. Traffic center of four points in a plane. 11. Traffic network for four points. 12. Unfold and straighten. 13. Billiards. 14. Geophysical exploration. 23. Shortest lines on a polyhedral surface. 24. Shortest lines (geodesics) on a curved surface. 26. A construction by paper-folding. 27. The die is cast. 28. The Deluge. 29. Not so deep as a well. 30. A useful extreme case. 32. The Calculus of Variations. 33. From the equilibrium of cross-sections to the equilibrium of the solids. 38. Archimedes' Method in retrospect.]

## CHAPTER X. THE ISOPERIMETRIC PROBLEM

168

1. Descartes' inductive reasons. 2. Latent reasons. 3. Physical reasons. 4. Lord Rayleigh's inductive reasons. 5. Deriving consequences. 6. Verifying consequences. 7. Very close. 8. Three forms of the Isoperimetric Theorem. 9. Applications and questions

Examples and Comments on Chapter X, 1-43; [First Part, 1-15; Second Part, 16-43]. [1. Looking back. 2. Could you derive some part of the result differently? 3. Restate with more detail. 7. Can you use the method for some other problem? 8. Sharper form of the Isoperimetric Theorem. 16. The stick and

the string. 21. Two sticks and two strings. 25. Dido's problem in solid geometry. 27. Bisectors of a plane region. 34. Bisectors of a closed surface. 40. A figure of many perfections. 41. An analogous case. 42. The regular solids. 43. Inductive reasons.]

CHAPTER XI. FURTHER KINDS OF PLAUSIBLE REASONS 190

1. Conjectures and conjectures. 2. Judging by a related case. 3. Judging by the general case. 4. Preferring the simpler conjecture. 5. Background. 6. Inexhaustible. 7. Usual heuristic assumptions

Examples and Comments on Chapter XI, 1-23. [16. The general case. 19. No idea is really bad. 20. Some usual heuristic assumptions. 21. Optimism rewarded. 23. Numerical computation and the engineer.]

FINAL REMARK 210

SOLUTIONS TO PROBLEMS 213

BIBLIOGRAPHY 279