
CONTENTS

Chapter I. 'NOT' and 'IF'

I

1. Symbols and sentences · 2 / EXERCISES · 7
2. From symbols to English and back · 8 / EXERCISES · 12
3. Derivability and validity of symbolic arguments · 13 / EXERCISES, GROUP I · 27 / EXERCISES, GROUP II · 32
4. Validity of English arguments · 32 / EXERCISES · 34
5. Fallacies · 35
6. Theorems · 40 / EXERCISES, GROUP I · 43 / EXERCISES, GROUP II · 44
7. Historical remarks · 44
8. Appendix: solutions to selected exercises · 46

Chapter II. 'AND', 'OR', 'IF AND ONLY IF'

50

1. Symbols and sentences · 50 / EXERCISES · 52
2. Translation and symbolization · 53 / EXERCISES · 57
3. Inference rules and derivability · 59 / EXERCISES · 63
4. Theorems with unabbreviated proofs · 63 / EXERCISES · 68
5. Abbreviated derivations · 71 / EXERCISES · 75
6. Theorems with abbreviated proofs · 75 / EXERCISES · 82
7. Arguments · 86 / EXERCISES, GROUP I · 87 / EXERCISES, GROUP II · 87
8. Truth-value analysis of sentences · 87 / EXERCISES, GROUP I · 92 / EXERCISES, GROUP II · 95
9. Truth-value analysis of arguments · 99 / EXERCISES, GROUP I · 100 / EXERCISES, GROUP II · 101 / EXERCISES, GROUP III · 103
10. Historical remarks · 105
11. Appendix: list of theorems of chapters I and II · 107
12. Appendix: solutions to selected exercises · 112

Chapter III. 'ALL' and 'SOME'

117

1. Variables, quantifiers, formulas · 117 / EXERCISES · 121
2. Bondage and freedom · 123 / EXERCISES · 125
3. Informal notational conventions · 126
4. Translation and symbolization · 126 / EXERCISES, GROUP I · 132 / EXERCISES, GROUP II · 133 / EXERCISES, GROUP III · 137
5. Inference rules · 139 / EXERCISES · 141
6. Derivations · 143 / EXERCISES · 145
7. Theorems with unabbreviated proofs · 146 / EXERCISES · 149
8. Abbreviated derivations · 149 / EXERCISES · 153
9. Fallacies · 153 / EXERCISES · 159
10. Theorems with abbreviated proofs · 161 / EXERCISES · 169
11. Arguments · 170 / EXERCISES, GROUP I · 171 / EXERCISES, GROUP II · 172 / EXERCISES, GROUP III · 173
12. Invalidity: truth-functional expansions and models · 174 / EXERCISES, GROUP I · 180 / EXERCISES, GROUP II · 182 / EXERCISES, GROUP III · 182 / EXERCISES, GROUP IV · 183
13. Historical remarks · 184
14. Appendix: list of theorems of chapter III · 185
15. Appendix: solutions to selected exercises · 187
16. Appendix: summary of the system of logic developed in chapters I-III · 197

Chapter IV. 'ALL' and 'SOME', *continued*

201

1. Terms and formulas · 201 / EXERCISES · 206
2. Bondage and freedom · 206 / EXERCISES · 207
3. Informal notational conventions · 208 / EXERCISES · 208
4. Translation and symbolization · 209 / EXERCISES, GROUP I · 213 / EXERCISES, GROUP II · 215 / EXERCISES, GROUP III · 216 / EXERCISES, GROUP IV · 218
5. Revised inference rules · 219 / EXERCISES, GROUP I · 221 / EXERCISES, GROUP II · 222 / EXERCISES, GROUP III · 222
6. Theorems · 222 / EXERCISES · 228
7. Invalidity: truth-functional expansions and models · 228 / EXERCISES · 236
8. Invalidity: false arithmetical translations · 237 / EXERCISES, GROUP I · 245 / EXERCISES, GROUP II · 246

- 9. Arguments of English · 246 / EXERCISES, GROUP I · 247 /
EXERCISES, GROUP II · 248 / EXERCISES, GROUP III · 248 /
EXERCISES, GROUP IV · 249
- 10. Historical remarks · 250
- 11. Appendix: list of theorems of chapter IV · 250
- 12. Appendix: solutions to selected exercises · 251

Chapter V. 'IS' (in one sense) 263

- 1. Terms and formulas · 263
- 2. Translation and symbolization · 264 / EXERCISES, GROUP I ·
267 / EXERCISES, GROUP II · 268
- 3. Inference rules · 269 / EXERCISES · 274
- 4. Theorems · 275 / EXERCISES, GROUP I · 281 / EXERCISES,
GROUP II · 282 / EXERCISES, GROUP III · 283 / EXERCISES,
GROUP IV · 284
- 5. Invalidity · 285 / EXERCISES, GROUP I · 290 / EXERCISES,
GROUP II · 290 / EXERCISES, GROUP III · 291 / EXERCISES,
GROUP IV · 291 / EXERCISES, GROUP V · 292 / EXERCISES,
GROUP VI · 293 / EXERCISES, GROUP VII · 294
- 6. Historical remarks · 295
- 7. Appendix: list of theorems of chapter V · 296
- 8. Appendix: solutions to selected exercises · 297

Chapter VI. 'THE' 306

- 1. Descriptive phrases, terms, formulas · 306 / EXERCISES · 311
- 2. Bondage and freedom · 311 / EXERCISES · 313
- 3. Informal notational conventions · 314 / EXERCISES · 314
- 4. Translation and symbolization · 315 / EXERCISES · 317
- 5. Inference rules · 318 / EXERCISES · 320
- 6. Theorems · 321 / EXERCISES · 327
- 7. Invalidity · 327 / EXERCISES · 337
- 8. Arguments of English · 339 / EXERCISES, GROUP I · 340 /
EXERCISES, GROUP II · 340
- 9. Historical remarks · 341
- 10. Appendix: list of theorems of chapter VI · 342
- 11. Appendix: solutions to selected exercises · 343

Chapter VII. ADDITIONAL DERIVATIONAL PROCEDURES

346

1. Alphabetic variance · 346 / EXERCISES · 348
2. Substitution · 349 / EXERCISES · 355
3. Abbreviated derivations reconsidered; biconditional derivations · 360 / EXERCISES · 368
4. Formulas without overlay and prenex normal form, again · 371 / EXERCISES · 373
5. Abbreviated derivations justified; generalized principles of interchange · 375 / EXERCISES · 379
6. Historical remarks · 381
7. Appendix: list of theorems of chapter VII · 381
8. Appendix: solutions to selected exercises · 381
9. Appendix: summary of the system of logic developed in chapters I–VII · 386

Chapter VIII. 'THE' again: A RUSSELLIAN THEORY OF DESCRIPTIONS

392

1. The Fregean theory of chapter VI reconsidered · 392 / EXERCISES · 394
2. A Russellian theory of descriptions · 394 / EXERCISES · 407
3. Historical remarks · 407
4. Appendix: list of theorems of chapter VIII · 408
5. Appendix: solutions to selected exercises · 409
6. Appendix: summary of the Russellian system of logic developed in Chapter VIII · 410

Chapter IX. AUTOMATIC PROCEDURES

411

1. Introduction · 411
2. Tautologies reconsidered · 411 / EXERCISES · 418
3. Tautological implication; generalized indirect derivation · 418
4. A proof procedure for prenex formulas · 419 / EXERCISES · 424
5. A derivation procedure for prenex arguments · 425 / EXERCISES · 426
6. Conversion to prenex form · 427 / EXERCISES · 430
7. A derivation procedure for arbitrary symbolic arguments · 430 / EXERCISES · 432
8. A decision procedure for certain prenex formulas · 433 / EXERCISES · 435

- 9. A decision procedure for certain prenex arguments · 435 /
 EXERCISES · 436
- 10. Historical remarks · 437

Chapter X. DEFINITIONS: FORMAL THEORIES 438

- 1. The vocabulary of formal languages · 438
- 2. Bondage and freedom; proper substitution; alphabetic variance · 441
- 3. Informal notational conventions · 444
- 4. Derivability · 445
- 5. Formal theories; the theory of commutative ordered fields · 446
- 6. Extensions of theories; the theory of real numbers · 452
- 7. Definitions · 461

Chapter XI. VARIABLE-BINDING OPERATORS 475

- 1. Definitions reconsidered · 475
- 2. The theory of convergence · 477
- 3. A sketch of further developments · 491
- 4. Historical remarks · 497
- 5. Appendix: list of constants used in chapters X and XI · 498
- 6. Appendix: list of axioms, definitions, and theorems of chapters X and XI · 499

BIBLIOGRAPHY 509

INDEX OF PROPER NAMES 515

INDEX OF SUBJECTS 517