# TABLE OF CONTENTS

## Part I: NATURAL DEDUCTION SYSTEMS

## **/3** Chapter 1: Validity

1.1	Arguments	3
1.2	Necessarily true statements	6
1.3	Valid logical formulas	9
1.4	Formal language	13
1.5	The use/mention distinction	14

## **20** Chapter 2: Sentential Language

9.1	Conjunction and danial	16
2.1	Conjunction and demai	10
2.2	Other statement connectives	19
2.3	Determining the truth-value of compound statements	21
<b>2.4</b>	Statement connectives and the natural language	<b>24</b>
2.5	Putting the natural language into symbols	<b>28</b>
<b>2.6</b>	Formation rules for the sentential language	31

# 32 Chapter 3: Decision Procedures

3.1	Truth-tables	36
<b>3.2</b>	Valid, inconsistent, and contingent formulas	38
3.3	Arguments and truth-tables	40
<b>3.4</b>	A short-cut truth-table method	45
3.5	Logical equivalence and transformations	47
3.6	Other two-place operators	54
3.7	Normal forms and testing for validity	57
	Supplement: Logical networks	61

# **34** Chapter 4: Natural Deduction System SC

4.1	Rules of inference and proofs	68
4.2	Four additional rules of inference	72
4.3	Assumption discharging rules	76
4.4	Useful strategies	83
4.5	System SC	86
4.6	Some theorems of SC	89
4.7	Two derived rules	91
4.8	Soundness, consistency, and completeness of SC	97

#### 32 Chapter 5: Predicate Language

5.1	Predicates	102
5.2	Quantifiers	106
5.3	Interpretation of quantifiers	110
5.4	Valid predicate formulas	113
5.5	Proving the invalidity of predicate formulas	116
5.6	Proving the validity of predicate formulas	118
5.7	Polyadic predicates	120
5.8	Formation rules for the predicate language	128

## **26** Chapter 6: Natural Deduction System PC

6.1	Individual constants and tautologies	134
6.2	Universal elimination and existential introduction	136
6.3	Existential elimination and universal introduction	140
6.4	Strategies in using quantification rules	147
6.5	System PC	150
6.6	Theorems of PC	152
6.7	Soundness, consistency, and completeness of PC	155

## **2.2** Chapter 7: Predicate Logic with Identity

7.1	Identity	160
7.2	Some properties of two-place predicates	162
7.3	Predicate logic with identity: System PCI	166
7.4	Proofs for arguments	169
7.5	Symbolizing using the identity sign	171
7.6	Definite descriptions	175

#### Part II: LOGICAL AXIOMATIC SYSTEMS

#### 18 Chapter 8: Formal Axiom Systems

8.1	The development of geometry	186
8.2	Formal axiomatic theories	187
8.3	System CS	190
8.4	Metalogical properties of CS	194
8.5	Axiom systems and logic	199

#### **23** Chapter 9: Axioms for the Sentential Calculus

9.1	Preliminary discussion of PM	204
9.2	System PM	206
9.3	Development of PM	207
9.4	Derived rules	210
9.5	Consistency and soundness of PM	214
9.6	Independence of the axioms of PM	217
9.7	Completeness of PM	220
9.8	Completeness of SC	224

# 18 Chapter 10: Axioms for the Predicate Calculus with Identity

10.1	Preliminary discussion of LPC	227
10.2	System LPC	230
10.3	Development of LPC	232
10.4	LPC with identity	236
10.5	Consistency of LPC with identity	238
10.6	Soundness of LPC with identity	239
10.7	Completeness of predicate logic theories	242

#### Further readings

Index

248

245