CONTENTS

Acknowledgements IX
Interdependence scheme for the chapters XIV
Introduction XV
Recommended reading XIX

CHAPTER 0. PREREQUISITES 1

CHAPTER 1. BEGINNING MATHEMATICAL LOGIC 5

§1. General considerations 5
§2. Structures and formal languages 9
§3. Higher-order languages 14
§4. Basic syntax 15
§5. Notational conventions 18
§6. Propositional semantics 20
§7. Propositional tableaux 25
§8. The Elimination Theorem for propositional tableaux 31
§9. Completeness of propositional tableaux 33
§10. The propositional calculus 34
§11. The propositional calculus and tableaux 40
§12. Weak completeness of the propositional calculus 43
§13. Strong completeness of the propositional calculus 43
§14. Propositional logic based on $\neg$ and $\wedge$ 46
§15. Propositional logic based on $\neg$, $\rightarrow$, $\wedge$ and $\vee$ 47
§16. Historical and bibliographical remarks 48

CHAPTER 2. FIRST-ORDER LOGIC 49

§1. First-order semantics 49
§2. Freedom and bondage 54
§3. Substitution 57
§4. First-order tableaux 67
§5. Some "book-keeping" lemmas 72
§6. The Elimination Theorem for first-order tableaux 79
§7. Hintikka sets 83
§8. Completeness of first-order tableaux 88
§9. Prenex and Skolem forms 93
§10. Elimination of function symbols 97

§11. Elimination of equality 101
§12. Relativization 102
§13. Virtual terms 104
§14. Historical and bibliographical remarks 107

CHAPTER 3. FIRST-ORDER LOGIC (CONTINUED) 108
§1. The first-order predicate calculus 108
§2. The first-order predicate calculus and tableaux 115
§3. Completeness of the first-order predicate calculus 117
§4. First-order logic based on ∃ 122
§5. What have we achieved? 122
§6. Historical and bibliographical remarks 124

CHAPTER 4. BOOLEAN ALGEBRAS 125
§1. Lattices 125
§2. Boolean algebras 129
§3. Filters and homomorphisms 133
§4. The Stone Representation Theorem 141
§5. Atoms 150
§6. Duality for homomorphisms and continuous mappings 153
§7. The Rasiowa–Sikorski Theorem 157
§8. Historical and bibliographical remarks 159

CHAPTER 5. MODEL THEORY 161
§1. Basic ideas of model theory 161
§2. The Löwenheim–Skolem Theorems 168
§3. Ultraproducts 174
§4. Completeness and categoricity 184
§5. Lindenbaum algebras 191
§6. Element types and $\aleph_0$-categoricity 203
§7. Indiscernibles and models with automorphisms 214
§8. Historical and bibliographical remarks 224

CHAPTER 6. RECURSION THEORY 226
§1. Basic notation and terminology 226
§2. Algorithmic functions and functionals 230
§3. The computer URIM 232
§4. Computable functionals and functions 237
§5. Recursive functionals and functions 239
§6. A stockpile of examples 247
§7. Church's Thesis 257
§8. Recursiveness of computable functionals 259
§9. Functionals with several sequence arguments 265
§10. Fundamental theorems 266
§11. Recursively enumerable sets 277
§12. Diophantine relations 284
§13. The Fibonacci sequence 288
§14. The power function 296

§15. Bounded universal quantification . . . 305
§16. The MRDP Theorem and Hilbert's Tenth Problem . . . 311
§17. Historical and bibliographical remarks . . . 314

CHAPTER 7. LOGIC — LIMITATIVE RESULTS . . . 316

§1. General notation and terminology . . . 316
§2. Nonstandard models of Ω . . . 318
§3. Arithmeticity . . . 324
§4. Tarski's Theorem . . . 327
§5. Axiomatic theories . . . 332
§6. Baby arithmetic . . . 334
§7. Junior arithmetic . . . 336
§8. A finitely axiomatized theory . . . 340
§9. First-order Peano arithmetic . . . 342
§10. Undecidability . . . 347
§11. Incompleteness . . . 353
§12. Historical and bibliographical remarks . . . 359

CHAPTER 8. RECURSION THEORY (CONTINUED) . . . 361

§1. The arithmetical hierarchy . . . 361
§2. A result concerning T_Ω . . . 369
§3. Encoded theories . . . 370
§4. Inseparable pairs of sets . . . 372
§5. Productive and creative sets; reducibility . . . 376
§6. One-one reducibility; recursive isomorphism . . . 384
§7. Turing degrees . . . 388
§8. Post's problem and its solution . . . 392
§9. Historical and bibliographical remarks . . . 398

CHAPTER 9. INTUITIONISTIC FIRST-ORDER LOGIC . . . 400

§1. Preliminary discussion . . . 400
§2. Philosophical remark . . . 403
§3. Constructive meaning of sentences . . . 403
§4. Constructive interpretations . . . 404
§5. Intuitionistic tableaux . . . 408
§6. Kripke's semantics . . . 416
§7. The Elimination Theorem for intuitionistic tableaux . . . 422
§8. Intuitionistic propositional calculus . . . 433
§9. Intuitionistic predicate calculus . . . 434
§10. Completeness . . . 438
§11. Translations from classical to intuitionistic logic . . . 442
§12. The Interpolation Theorem . . . 445
§13. Some results in classical logic . . . 452
§14. Historical and bibliographical remarks . . . 457

CHAPTER 10. AXIOMATIC SET THEORY . . . 459

§1. Basic developments . . . 459
§2. Ordinals . . . 468

§3. The Axiom of Regularity 477
§4. Cardinality and the Axiom of Choice 487
§5. Reflection Principles 491
§6. The formalization of satisfaction 497
§7. Absoluteness 502
§8. Constructible sets 509
§9. The consistency of **AC** and **GCH** 516
§10. Problems 522
§11. Historical and bibliographical remarks 529

CHAPTER 11. NONSTANDARD ANALYSIS 531
§1. Enlargements 532
§2. Zermelo structures and their enlargements 536
§3. Filters and monads 543
§4. Topology 553
§5. Topological groups 561
§6. The real numbers 566
§7. A methodological discussion 572
§8. Historical and bibliographical remarks 573

BIBLIOGRAPHY 576

GENERAL INDEX 584

INDEX OF SYMBOLS 595