Table of Contents

Preface ix

Introduction 1

1 The Subject Matter 1
2 Goals and Organization 4
3 Assumed Background Knowledge 7
4 The Presentation 14

Chapter 1. Sentential Calculus 17

1.1 Syntax of the Sentential Calculus 18
1.2 Correspondence to Natural Languages 23
1.3 Semantics of the Sentential Calculus 30
1.4 Some Metatheoretical Concepts 41
1.5 Principles for Sentential Calculus Derivations 56
1.6 Adequacy and Use of Sentential Calculus Derivation Rules 72
1.6.1 Soundness and Completeness 72
1.6.2 Strategies for Constructing Proofs 79
1.6.3 Sentential Calculus Derivation Examples 86
1.7 Conjunctive Normal Form and Resolution Proofs 94

Chapter 2. Basic Set Theory 103

2.1 Sets 103
2.1.1 Extensionality, Predicates, and Abstraction 103
2.1.2 Some Special Sets and Set Operations 111
2.2 Relations 123
2.2.1 General Features 123
2.2.2 Special Kinds of Relations 134
2.3 Functions 154
2.3.1 Basic Ideas 154

2.3.2 Compositions and Inverses 161
2.4 Relational Systems 172

Chapter 3. Recursion and Mathematical Induction 179

3.1 The Natural Number System 179
3.1.1 Introduction 179
3.1.2 Peano's Axioms and the Induction Principle 183
3.1.3 Definition by Recursion 188
3.2 Basic Arithmetic 196
3.2.1 Some Simple Functions 196
3.2.2 Additional Arithmetical Definitions 201
3.3 Extensions of Recursive Definition and Induction 214
3.3.1 Some Additional Applications of the Recursion Theorem 214
3.3.2 The Well-Ordering of the Natural Numbers 219
3.3.3 Course of Values Induction 229
3.3.4 Two Arithmetical Algorithms 232
3.3.5 Pitfalls of Recursion 239
3.4 Non-Numerical Data 248
3.4.1 Strings 249
3.4.2 A Simple Treatment of Lists 255
3.4.3 Sentential Calculus Expressions 266
3.4.4 Stacks and Queues 268

Chapter 4. Predicate Calculus 283

4.1 Syntax of the Predicate Calculus 285
4.2 Semantical Aspects of the Predicate Calculus 293
4.2.1 Interpretations and Truth 293
4.2.2 Tautologous Sentences in Predicate Calculus 299
4.2.3 Tautological Consequences in Predicate Calculus 302
4.3 Predicate Calculus Derivations 307
4.3.1 Derivation Rules 307
4.3.2 Proof Strategies and Examples 316
4.3.3 Adequacy of the Predicate Calculus Rules 322
4.4 Application Example 332
4.5 Identity and Function Symbols 340
4.5.1 Extension of the Syntax 340
4.5.2 Semantics of Predicate Calculus with Identity and Function Symbols 343

4.5.3 Derivation Rules for Predicate Calculus with Identity and Function Symbols 345
4.5.4 Use of Identity in Representing Information 347
4.6 Formalized Theories 352

References 359

Answers to Selected Exercises 365

Chapter 1 365
Chapter 2 369
Chapter 3 374
Chapter 4 382

The Greek Alphabet 389

Glossary of Symbols 391

Index 395