

# CONTENTS

	PAGE
<i>Preface</i>	v
<i>Acknowledgments</i>	viii
<i>Preface to Revised Edition</i>	ix
<i>Introduction</i>	I
<b>CHAPTER ONE. <i>Statements</i></b>	<b>9</b>
1. Conjunction, Alternation, and Denial	11
2. The Conditional	14
3. Iterated Composition	18
4. Use versus Mention	23
5. Statements about Statements	27
6. Quasi-Quotation	33
7. Parentheses and Dots	37
8. Reduction to Three Primitives	42
9. Reduction to One Primitive	45
10. Tautology	50
11. Selected Tautologous Forms	55
<b>CHAPTER TWO. <i>Quantification</i></b>	<b>63</b>
12. The Quantifier	65
13. Formulae	71
14. Bondage, Freedom, Closure	76
15. Axioms of Quantification	80
16. Theorems	85
17. Metatheorems	89
18. Substitutivity of the Biconditional	96
19. Existential Quantification	101
20. Distribution of Quantifiers	105
21. Alphabetic Variance	109
<b>CHAPTER THREE. <i>Terms</i></b>	<b>117</b>
22. Class and Member	119
23. Logical Formulae	123

## CONTENTS

xi

	PAGE
24. Abstraction . . . . .	128
25. Identity . . . . .	134
26. Abstraction Resumed . . . . .	140
27. Descriptions and Names . . . . .	146
<b>CHAPTER FOUR. <i>Extended Theory of Classes</i></b> . . . . .	<b>153</b>
28. Stratification . . . . .	155
29. Further Axioms of Membership . . . . .	160
30. Substitutivity of Identity . . . . .	167
31. Substitution for Variables . . . . .	170
32. Further Consequences . . . . .	175
33. Logical Product, Sum, Complement . . . . .	179
34. Inclusion . . . . .	185
35. Unit Classes . . . . .	189
<b>CHAPTER FIVE. <i>Relations</i></b> . . . . .	<b>195</b>
36. Pairs and Relations . . . . .	197
37. Abstraction of Relations . . . . .	202
38. Converse, Image, Relative Product . . . . .	208
39. The Ancestral . . . . .	215
40. Functions . . . . .	221
41. Abstraction of Functions . . . . .	225
42. Identity and Membership as Relations . . . . .	229
<b>CHAPTER SIX. <i>Number</i></b> . . . . .	<b>235</b>
43. Zero, One, Successor . . . . .	237
44. Natural Numbers . . . . .	241
45. Counter Sets . . . . .	246
46. Finite and Infinite . . . . .	250
47. Powers of Relations . . . . .	253
48. Arithmetical Sum, Product, Power . . . . .	259
49. Familiar Identities of Arithmetic . . . . .	262
50. Ratios . . . . .	266
51. Real Numbers . . . . .	271
52. Further Extensions . . . . .	275
<b>CHAPTER SEVEN. <i>Syntax</i></b> . . . . .	<b>281</b>
53. Formality . . . . .	283

	PAGE
54. The Syntactical Primitive . . . . .	287
55. Protosyntax . . . . .	291
56. Formula and Matrix Defined . . . . .	295
57. Axioms of Quantification Defined . . . . .	299
58. Theorem Defined . . . . .	302
59. Protosyntax Self-Applied . . . . .	306
60. Incompleteness . . . . .	310
 <b>APPENDIX. Theorem versus Metatheorem</b> . . . . .	 319
<i>List of Definitions</i> . . . . .	323
<i>List of Theorems and Metatheorems</i> . . . . .	325
<i>Bibliographical References</i> . . . . .	331
<i>Index of Proper Names</i> . . . . .	339
<i>Index of Subjects</i> . . . . .	341