CONTENTS OF VOLUME ONE

ACKNOWLEDGEMENTS

1. INTRODUCTION

For the reader's comfort. The beginning of modern mathematics. Are mathematicians human? Witless parodies. Illimitable scope of mathematical evolution. Pioneers and scouts. A clue through the maze. Continuity and discreteness. Remarkable rarity of common sense. Vivid mathematics or vague mysticism? Four great ages of mathematics. Our own the Golden Age.

2. MODERN MINDS IN ANCIENT BODIES ZENO (fifth century B.C.), EUDOXUS (408-355 B.C.) Archimedes (287?-212 B.C.)

Modern ancients and ancient moderns. Pythagoras, great mystic, greater mathematician. Proof or intuition? The taproot of modern analysis. A bumpkin upsets the philosophers. Zeno's unresolved riddles. Plato's needy young friend. Inexhaustible exhaustion. The useful conics. Archimedes, aristocrat, greatest scientist of antiquity. Legends of his life and personality. His discoveries and claim to modernity: A sturdy Roman. Defeat of Archimedes and triumph of Rome.

3. GENTLEMAN, SOLDIER, AND MATHEMA-TICIAN

DESCARTES (1596-1650)

The good old days. A child philosopher but no prig. Inestimable advantages of lying in bed. Invigorating doubts. Peace in war. Converted by a nightmare. Revelation of analytic geometry. More butchering. Circuses, professional jealousy, swashbuckling, accommodating lady friends. Distaste for hell-fire and respect for the Church. Saved by a brace of cardinals. A Pope brains himself. Twenty years vii

1

19

a recluse. The Method. Betrayed by fame. Doting Elisabeth. What Descartes really thought of her. Conceited Christine. What she did to Descartes. Creative simplicity of his geometry.

4. THE PRINCE OF AMATEURS

FERMAT (1601-65)

Greatest mathematician of the seventeenth century. Fermat's busy, practical life. Mathematics his hobby. His flick to the calculus. His profound physical principle. Analytic geometry again. Arithmetica and logistica. Fermat's supremacy in arithmetic. An unsolved problem on primes. Why are some theorems 'important'? An intelligence test. 'Infinite descent.' Fermat's unanswered challenge to posterity.

5. 'GREATNESS AND MISERY OF MAN'

PASCAL (1623-62)

An infant prodigy buries his talent. At seventeen a great geometer. Pascal's wonderful theorem. Vile health and religious inebriety. The first calculating Frankenstein. Pascal's brilliance in physics. Holy sister Jacqueline, soul-saver. Wine and women? 'Get thee to a nunnery!' Converted on a spree. Literature prostituted to bigotry. The Helen of Geometry. A celestial toothache. What the post-mortem revealed. A gambler makes mathematical history. Scope of the theory of probability. Pascal creates the theory with Fermat. Folly of betting against God or the Devil.

6. ON THE SEASHORE

NEWTON (1642-1727)

Newton's estimate of himself. An uncertified youthful genius. Chaos of his times. On the shoulders of giants. His one attachment. Cambridge days. Young Newton masters futility of suffering fools gladly. The Great Plague a greater blessing. Immortal at twenty-four (or less). The calculus. Newton unsurpassed in pure mathematics, supreme in natural philosophy. Gnats, hornets, and exasperation. The Principia. Samuel Pepys and other fussers. The flattest anticlimax in history. Controversy, theology, chronology, alchemy, public office, death.

7. MASTER OF ALL TRADES

LEIBNIZ (1646-1716)

Two superb contributions. A politician's offspring. Genius at fifteen. Seduced by the law. The 'universal characteristic.' Symbolic reasoning. Sold out to ambition. A master diplomat. Diplomacy being what it is, the diplomatic exploits of the master are left to the historians. Fox into historian, statesman into mathematician. Applied ethics. Existence of God. Optimism. Forty years of futility. Discarded like a dirty rag.

8. NATURE OR NURTURE?

THE BERNOULLIS (seventeenth and eighteenth centuries)

Eight mathematicians in three generations. Clinical evidence for heredity. The calculus of variations.

9. ANALYSIS INCARNATE

EULER (1707-83)

The most prolific mathematician in history. Snatched from theology. Rulers foot the bills. Practicality of the unpractical. Celestial mechanics and naval warfare. A mathematician by chance and foreordination. Trapped in St Petersburg. The virtues of silence. Half blind in his morning. Flight to liberal Prussia. Generosity and boorishness of Frederick the Great. Return to hospitable Russia. Generosity and graciousness of Catherine the Great. Total blindness at noon. Master and inspirer of masters for a century.

10. A LOFTY PYRAMID

LAGRANGE (1736-1813)

Greatest and most modest mathematician of the eighteenth century. Financial ruin his opportunity. Conceives his masterpiece at nincteen. Magnanimity of Euler. Turin, to Paris, to Berlin: a grateful bastard aids a genius. Conquests in celestial mechanics. Frederick the Great condescends. Absent-minded marriage. Work as a vice. A classic in arithmetic. The Mécanique analytique a living masterpiece. A landmark in the theory of equations. Welcomed 143

xi

in Paris by Marie Antoinette. Nervous exhaustion, melancholia, and universal disgust in middle life. Reavakened by the French Revolution and a young girl. What Lagrange thought of the Revolution. The metric system. What the revolutionists thought of Lagrange. How a philosopher dies.

11. FROM PEASANT TO SNOB

LAPLACE (1749-1827)

Humble as Lincoln, proud as Lucifer. A chilly reception and a warm welcome. Laplace grandiosely attacks the solar system. The Mécanique céleste. His estimate of himself. What others have thought of him. The 'potential' fundamental in physics. Laplace in the French Revolution. Intimacy with Napoleon. Laplace's political realism superior to Napoleon's.

12. FRIENDS OF AN EMPEROR

MONGE (1746-1818), FOURIER (1768-1830)

A knife grinder's son and a tailor's boy help Napoleon to upset the aristocrat's applecart. Comic opera in Egypt. Monge's descriptive geometry and the Machine Age. Fourier's analysis and modern physics. Imbecility of trusting in princes or proletarians. Boring to death and bored to death.

13. THE DAY OF GLORY

PONCELET (1788-1867)

Resurrected from a Napoleonic shambles. The path of glory leads to jail. Wintering in Russia in 1812. What genius does in prison. Two years of geometry in hell. The rewards of genius: stupidities of routine. Poncelet's projective geometry. Principles of continuity and duality.

14. THE PRINCE OF MATHEMATICIANS

GAUSS (1777–1855)

Gauss the mathematical peer of Archimedes and Newton. Humble origin. Paternal brutality. Unequalled intellectual precocity. His chance, at ten. By twelve he dreams revolutionary discoveries, by eighteen achieves them. The Disquisitiones Arithmeticae. Other epochal works summarized. 18

22

20

The Ceres disaster. Napoleon, indirectly robbing Gauss, takes second best. Fundamental advances in all branches of mathematics due to Gauss too numerous for citation: see the account given. A sage of sages. Unwelcome death.

15. MATHEMATICS AND WINDMILLS

САИСНУ (1789-1857)

Change in nature of mathematics with nineteenth century. Childhood in the French Revolution. Cauchy's early miseducation. Lagrange's prophecy. The young Christian engineer. Prophetic acuteness of Malus. The theory of groups. In the front rank at twenty-seven. One of Fermat's enigmas solved. The pious hippopotamus. Butted by Charles the Goat. Memoirs on astronomy and mathematical physics. Sweetness and obstinacy invincible. The French Government makes a fool of itself. Cauchy's place in mathematics. Drawbacks of an irreproachable character.

A complete index to both volumes will be found at the end of Volume Two

CONTENTS OF VOLUME TWO

PREFACE

. THE COPERNICUS OF GEOMETRY

LOBATCHEWSKY (1793-1856)

The widow's mite. Kazan. Appointed professor and spy. Universal ability. Lobatchewsky as an administrator. Reason and incense combat the cholera. Russian gratitude. Humiliated in his prime. Blind as Milton, Lobatchewsky dictates his masterpiece. His advance beyond Euclid. Non-Euclidean geometry. A Copernicus of the intellect.

. GENIUS AND POVERTY

ABEL (1802-29)

Norway in 1802. Smothered by clerical fecundity. Abel's awakening. Generosity of a teacher. A pupil of the masters. His lucky blunder. Abel and the quintic. The Government to the rescue. Abel's grand tour of mathematical Europe not so grand. French civility and German cordiality. Crelle and his Journal. Cauchy's unpardonable sin. 'Abel's Theorem.' Something to keep mathematicians busy 500 years. Crowning a corpse.

. THE GREAT ALGORIST

JACOBI (1804-51)

Galvanoplastics versus mathematics. Born rich. Jacobi's philological ability. Dedicates himself to mathematics. Early work. Cleaned out. A goose among foxes. Hard times. Elliptic functions. Their place in the general development. Inversion. Work in arithmetic, dynamics, algebra, and Abelian functions. Fourier's pontification. Jacobi's retort.

AN IRISH TRAGEDY

HAMILTON (1805-65)

Ireland's greatest. Elaborate miseducation. Discoveries at seventeen. A unique university career. Disappointed in love. Hamilton and the poets. Appointed at Dunsink.

xxiii

323

337

359

Systems of rays. The Principia of optics. Prediction of conical refraction. Marriage and alcohol. Fields. Complex numbers. The commutative law repealed. Quarternions. Mountains of paper.

20. GENIUS AND STUPIDITY

GALOIS (1811-32)

An all-time world record in stupidity. Galois' childhood. The pedagogues surpass themselves. At sixteen Galois repeats Abel's mistake. Politics and education. Examinations as arbiters of genius. Hounded to death by a priest. More academic ineptitude. Absent-minded Cauchy again. Driven to rebellion. A master mathematician at nineteen. 'A carcase to stir up the people.' The foulest sewer in Paris. Patriots rush to the field of honour. Galois' last night. The riddle of equations solved. Buried like a dog.

21. INVARIANT TWINS

SYLVESTER (1814-97); CAYLEY (1821-95)

Cayley's contributions. Early life. Cambridge. Recreations. Called to the Bar. Fourteen years in the law. Cayley meets his collaborator. Sylvester's stormier life. Hamstrung by religion. Cayley and Sylvester contrasted. Sylvester's mission to the Virginians. Further false steps. The theory of invariants. Called to Johns Hopkins University. Inextinguishable vitality. 'Rosalind.' Cayley's unification of geometry. Space of n dimensions. Matrices. Oxford endorses Sylvester. Respectable at last.

22. MASTER AND PUPIL

WEIERSTRASS (1815-97); SONJA KOWALEWSKI (1850-91)

The father of modern analysis. Relations of Weierstrass to his contemporaries. The penalties of brilliance. Forced into law, forces himself out. Beer and broadswords. A fresh start. Debt to Gudermann. Fifteen years in the mud. Miraculous extrication. Weierstrass' life problem. Too much success. Sonja storms the master. His favourite pupil. Their friendship. A woman's gratitude. Repenting, Sonja wins Paris prize. Weierstrass universally honoured. Power series. Arithmetization of analysis. Doubts.

XX

23. COMPLETE INDEPENDENCE BOOLE (1815-64)

British mathematics. Damned at birth by snobbery. Boole's struggle for education. False diagnoses. Providence intervenes. Discovery of invariants. What is algebra? A philosopher attacks a mathematician. Frightful carnage. Boole's chance. 'The Laws of Thought.' Symbolic logic. Its mathematical significance. Boolean algebra. Dead in his prime.

24. THE MAN, NOT THE METHOD HERMITE (1822–1901)

Old problems and new methods. Hermite's masterful mother. His detestation of examinations. Instructs himself. Higher mathematics sometimes easier than elementary. Educational disasters. Letters to Jacobi. A master at twenty-one. Revenge on his examiners. Abelian functions. Pestered by Cauchy. Hermite's mysticism. Solution of the general quintic. Transcendental numbers. A hint to circlesquarers. Hermite's internationalism.

25. THE DOUBTER

KRONECKER (1823-91)

Legend of an American saint. Lucky Kronecker. School triumphs. Great gifts. Algebraic numbers. Battles with Weierstrass. Kronecker's business career. Returns rich to mathematics. The Galois theory. Kronecker's lectures. His scepticism his most original contribution.

26. ANIMA CANDIDA

RIEMANN (1826–66)

Poor but happy. Riemann's chronic shyness. Destined for the church. Saved. A famous hypothesis. Career at Göttingen. 'A new mathematic.' Physical researches. Application of topology to analysis. Epoch-making essay on foundations of geometry. Gauss enthusiastic. The blessings of poverty. A root of tensor analysis. Quest for health. Under a fig tree. Riemann's landmark in geometry. Curvature of space. Pathbreaking for relativity.

27. ARITHMETIC THE SECOND KUMMER (1810-93); DEDEKIND (1831-1916)

Aged in the wood. Napoleonic warp to Kummer's geniality. Equally gifted in the abstract and the concrete. What 514

534

xxi

Fermat's Last Theorem started. Theory of ideal numbers. Kummer's invention comparable to Lobatchewsky's. Wave surface in four dimensions. Big of body, mind, and heart. Dedekind, last pupil of Gauss. First expositor of Galois. Early interest in science. Turns to mathematics. Dedekind's work on continuity. His creation of the theory of ideals.

28. THE LAST UNIVERSALIST POINCARÉ (1854–1912)

Poincaré's universality and methods. Childhood setbacks. Seized by mathematics. Keeps his sanity in Franco-Prussian war. Starts as mining engineer. First great work. Automorphic functions. 'The keys of the algebraic cosmos.' The problem of n bodies. Is Finland civilized? Poincaré's new methods in celestial mechanics. Cosmogony. How mathematical discoveries are made. Poincaré's account. Forebodings and premature death.

29. PARADISE LOST?

CANTOR (1845-1918)

Old foes with new faces. Rotting creeds. Cantor's artistic inheritance and father-fixation. Escape, but too late. His revolutionary work gets him nowhere. Academic pettiness. Disastrous consequences of 'safety first'. An epochal result. Paradox or truth? Infinite existence of transcendentals. Aggressiveness advances, timidity retires. Further spectacular claims. Two types of mathematicians. Insane? Counter-revolution. The battle grows fiercer. Cursing the enemy. Universal loss of temper. Where stands mathematics to-day? And where will it stand to-morrow? Invictus.

INDEX TO VOLUMES ONE AND TWO

64

58