

CONTENTS

CHAPTER I

HISTORICAL INTRODUCTION (p. 1)

CHAPTER II

THE SCOPE OF PROBABILITY

§ 1. <i>The meaning of chance</i>	12
Chance and scientific method	12
The three fields of study	15
Chance in scientific observation	16
§ 2. <i>On the definition of probability</i>	17
The definition of mathematical probability	17
Definition of statistical probability	18
<i>A priori</i> probability	19
Probability as a branch of logic	20
The principle of insufficient reason	24
Other definitions of probability	26
§ 3. <i>Mathematical determinism</i>	33
The typical problem of mathematics	34
The two classes of investigation	36
The nature of mathematical determinism	37

CHAPTER III

THE THEORY OF ARRANGEMENTS

The number of r -permutations of n different objects	41
The number of permutations of n objects which are not all different	42
The number of r -combinations of n different objects	43
Stirling's Theorem	44
The Binomial Theorem	45
The Binomial coefficients	45
Greatest term in the expansion	46
The Multinomial Theorem	46
The Binomial Series	47
The number of homogeneous products of degree r which can be formed with n letters	47

CHAPTER IV

ELEMENTARY THEOREMS ON MATHEMATICAL PROBABILITY

The addition theorem for probabilities	49
Mathematical expectation	50
Probability that a member of a class is not a member of a subclass	51

The multiplication theorem for probabilities	51
Tchebycheff's problem	53
Examples on Chapter IV	55

CHAPTER V

BERNOULLI'S THEOREM

§ 1. <i>Bernoulli's Theorem and its extensions</i>	58
Bernoulli's Theorem for a single subclass	58
The probability that not more than r members belong to a subclass	59
The probability that not less than r members belong to a subclass	60
Note on terminology	60
Greatest value of Bernoulli's probability	62
First generalization of Bernoulli's Theorem	62
The average value of r is also the most probable value	63
Case of probability varying from one trial to another	64
Second generalization of Bernoulli's Theorem	64
§ 2. <i>Bernoulli's Theorem and the Normal Law</i>	65
Proof of Stirling's Theorem	65
Approximate value of Bernoulli's probability when $p = \frac{1}{2}$	68
The Error Function	70
The general case	70
Examples on Chapter V	73

CHAPTER VI

EXTENSION TO CONTINUOUS DISTRIBUTIONS

Definition of probability for a continuous one-dimensional medium	75
Application to weighted probabilities	77
Extension to a continuous two-dimensional medium	78
Discrete and continuous entities	79
Probability that a characteristic is not present in a given sample	80
The Random Walk	81
Illustrative examples	83
Buffon's problem	86
Examples on Chapter VI	89

CHAPTER VII

THE THEORY OF ARRANGEMENTS (2)

Preliminary definitions	91
The number of arrangements of n different objects in r or fewer groups	91
The number of arrangements of n different objects in r groups	92
The case in which each group contains at least s objects	92
The number of arrangements of n identical objects in r parcels	93

CONTENTS

ix

The number of arrangements of n identical objects in r or fewer parcels	93
The case in which no parcel contains less than q objects	93
The number of arrangements of n different objects in r or fewer groups when not all the objects need be used	94
The number of arrangements of n different objects in r groups when not all the objects need be used	95
The number of arrangements in r indifferent groups	95
On the number of sets free from r specified letters	96
The number of total derangements of n objects	97
The number of arrangements of n different objects in r or fewer parcels	98
The number of arrangements of n different objects in r parcels	98
The number of arrangements of n identical objects in r parcels such that no parcel contains fewer than q or more than $q+t-1$ objects	98
Illustrative examples	99
Examples on Chapter VII	99

CHAPTER VIII

THE EMPIRICAL THEORY OF DISTRIBUTIONS

§ 1. <i>Hypothetical populations and typical constants</i>	102
Histograms	102
Frequency and probability curves	104
Empirical probability as a continuous function	105
On the meaning of 'population'	106
The application of statistical theory to physical measurement	106
Typical constants for a finite set of observations	110
Note on 'average' and 'mean'	112
Significance of the standard deviation	113
Definition of weights	115
Typical constants for a continuous distribution	115
Tchebycheff's Theorem.	117
§ 2. <i>The Gaussian Law</i>	118
First derivation of the Gaussian Law	118
Alternative derivation	121
Fundamental properties of the Error Function	124
The probable error	126
Applications of the Normal Law	128
Accuracy of the Arithmetic Mean	129
The Random Walk in two dimensions	131
The Gaussian Law and experiment	133
The significance of deviations	134
§ 3. <i>Other forms of hypothetical populations</i>	135
Derivation of a sample from a known population by a given selective process	135
The inverse problem	136

The Hermite polynomials	137
Standard deviation for Bernoullian populations	141
Bernoulli's Limit Theorem	142
Poisson distributions	142
The standard deviation for Poisson's Law	143
The telephone problem	144

CHAPTER IX

THE USE OF PROBABILITY IN SCIENTIFIC INDUCTION

§ 1. <i>The general problem</i>	146
Statement of the problem	146
Deduction of a sample from a given population of discontinuous type	147
Derivation of a particular population from a sample by a given selective process	148
Examples	149
The Bernoullian law of selection	151
Bayes's Theorem for a discontinuous distribution	152
Examples	153
Extension to continuous distributions	156
Applications	156
Two-dimensional distributions	160
§ 2. <i>The determination of a population from a given set of samples</i>	164
On the determination of hypothetical populations	164
The method of maximum likelihood	165
Applications	166
The method of least squares	169
Determination of the precision constant	170
Curve fitting	171
The line of regression	172
The coefficient of linear correlation	174
Parabolic correlation	175
The method of maximum correlation	176
Linear correlation in general	178
The Gaussian Law for two variables: correlation	180
Tests of significance for small samples	184
Other tests of significance	189
Examples on Chapter IX	194
APPENDIX: Table of the Error Function	198
INDEX	199