# PRINCIPIA MATHEMATICA

BY

#### ALFRED NORTH WHITEHEAD, Sc.D., F.R.S.

Fellow and late Lecturer of Trinity College, Cambridge

AND

BERTRAND RUSSELL, M.A., F.R.S.

Lecturer and late Fellow of Trinity College, Cambridge

VOLUME II



## CONTENTS OF VOLUME II

•

			PAGE
PREFATORY	STATEMENT OF SYMBOLIC CONVENTIONS .		ix
PART III.	CARDINAL ARITHMETIC.		
Summ	ary of Part III	• •	3
SECTION A	. DEFINITION AND LOGICAL PROPERTIES OF CARDINAL NUM	BERS .	4
<b>*100</b> .	Definition and elementary properties of cardinal numbers		13
<b>*101</b> .			19
*102.	On cardinal numbers of assigned types		24
*103.			36
<b>*104</b> .			42
*105.			52
*106.		· ·	60
- 200.		•••	
SECTION B.	Addition, Multiplication and Exponentiation .		66
*110	The arithmetical sum of two classes and of two cardinals		75
*111.	Double similarity		88
*112	The arithmetical sum of a class of classes		97
*113.			105
*110: *114.	The arithmetical product of a class of classes		124
*115.	Multiplicative classes and arithmetical classes	•	135
*116.		•••	143
*110. *117.		•	145
*117.		•	185
	General note on cardinal correlators	•	109
SECTION C.	FINITE AND INFINITE		187
*118.	Arithmetical substitution and uniform formal numbers		193
*119	Subtraction	•	201
*120.	Inductive cardinals		207
*120. *121.	Intervals.		233
*121. *122.	Progressions		253
*122.	N	•	268
*125.	Reflexive classes and cardinals	•	278
*125.			289
*125. *126.	On typically indefinite inductive cardinals	•	293
<b>#1</b> 40.	On typicany indentitie inductive cardinate	•	200

CONTENTS

PART	IV. I	RELATION-ARITHMETIC.				PAGE
	Sumn	nary of Part IV				301
SE		A. ORDINAL SIMILARITY AND RELATION-NUMBERS.				303
	*150.	Internal transformation of a relation				<b>30</b> 6
	*151.	Ordinal similarity				319
	<b>*152</b> .	Definition and elementary properties of relation-nur	nbers		•	<b>33</b> 0
	<b>*153</b> .	The relation-numbers $0_r$ , $2_r$ and $1_s$	•	•	•	334
	*154.	Relation-numbers of assigned types	•	·	·	339
	<b>*155</b> .	Homogeneous relation-numbers	•	•	•	344
Se	ECTION E	8. Addition of Relations, and the product of two	RELA	FION	3.	347
	*160.	The sum of two relations	•	•	•	351
	*161.	Addition of a term to a relation.	•	•	•	357 362
	*162. *163.	The sum of the relations of a field Relations of mutually exclusive relations	•	·	·	369
	*165. *164.	Double likeness		•	÷	376
	*165.	Relations of relations of couples				386
	<b>*166</b> .	The product of two relations				396
Se	CTION C	. THE PRINCIPLE OF FIRST DIFFERENCES, AND THE	NULTI	PLIC.	A-	
		TION AND EXPONENTIATION OF RELATIONS .		•		403
	<b>*170</b> .	On the relation of first differences among the sub-	lasse	s of	a	
		given class			•	411
	<b>*17</b> 1.	The principle of first differences (continued) .	•	•	•	423
	*172.	I	•	•	·	428
	*173. *17 <b>4</b> .	The product of the relations of a field (continued)		•	·	443 447
	*174. *176.	The associative law of relational multiplication . Exponentiation		•	•	458
	*177.	Propositions connecting $P_{df}$ with products and power			•	471
Se		ARITHMETIC OF RELATION-NUMBERS				473
	*180.			•		477
	*180. *181.	· · · · · · · · · · · · · · · · · · ·	•		•	482
	*182.	On separated relations.				487
	<b>*183</b> .	The sum of the relation-numbers of a field .		•		<b>496</b>
	<b>*184</b> .	The product of two relation-numbers	•	•	•	501
	*185.	The product of the relation-numbers of a field .	•	•	•	505
	<b>*186</b> .	Powers of relation-numbers	•	•	•	507
PART	V. SE	RIES.				
	Summ	ary of Part V.				513
SE	CTION A.	-				516
~	*200.		•	•	•	
	*200. *201.	Relations contained in diversity	•	•	·	518 525
	*201. *202.	Connected relations	•	•	•	ozo 533
	<b>*204</b> .	Elementary properties of series	•	•	•	547
	<b>*205</b> .	Maximum and minimum points		•	•	559
	<b>*206</b> .	Sequent points		•	•	577
	*207.	Limits	•	•	•	594
	*208.	The correlation of series			•	605

vi

#### CONTENTS

									PAGE
SECTION B.	ON SECTIONS, SEGMENTS, S	STRETCH	ES, AN	D Di	ERIVA	TIVES	•	•	612
<b>*210</b> .	On series of classes generate	ed by th	e rela	tion	of ind	lusio	n		615
<b>*211</b> .	On sections and segments								624
<b>*212</b> .	The series of segments								651
<b>*213</b> .	Sectional relations .								668
	Dedekindian relations								684
<b>*215</b> .	Stretches					•			691
<b>*216</b> .	Derivatives								700
<b>*217</b> .	On segments of sums and o	converse	۴.	•	•	•	•	•	710
SECTION C.	ON CONVERGENCE, AND TH	E LIMIT	s of :	Func	rions				715
<b>*230</b> .	On convergents								720
<b>*231</b> .	Limiting sections and ultim	ate oscil	lation	of a	fune	tion			727
<b>*232</b> ,	On the oscillation of a fund	tion as	the a	rgum	ent a	pproa	ches	a	
	given limit								737
<b>*233</b> .	On the limits of functions								745
<b>*234</b> .	Continuity of functions			•		•		•	753

### Available From Merchant Books

Principia Mathematica Volume I (ISBN 1-60386-182-3) Principia Mathematica Volume III (ISBN 1-60386-184-X) vii