The Birth of Model Theory

Löwenheim's Theorem in the Frame of the

Theory of Relatives

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Revised by the author

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