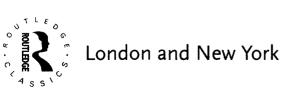
# **Bertrand** Russell

**Principles of Mathematics** 



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