

Covariant Loop Quantum Gravity

An Elementary Introduction to Quantum Gravity and Spinfoam Theory

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