

# Contents

PREFACE .....	<i>page</i> ix
INTRODUCTION .....	xii
Structural proof theory .....	xii
Use of this book in teaching .....	xiv
What is new in this book? .....	xvi
1. FROM NATURAL DEDUCTION TO SEQUENT CALCULUS .....	1
1.1. Logical systems .....	1
1.2. Natural deduction .....	5
1.3. From natural deduction to sequent calculus .....	13
1.4. The structure of proofs .....	20
Notes to Chapter 1 .....	23
2. SEQUENT CALCULUS FOR INTUITIONISTIC LOGIC .....	25
2.1. Constructive reasoning .....	25
2.2. Intuitionistic sequent calculus .....	28
2.3. Proof methods for admissibility .....	30
2.4. Admissibility of contraction and cut .....	33
2.5. Some consequences of cut elimination .....	40
Notes to Chapter 2 .....	46
3. SEQUENT CALCULUS FOR CLASSICAL LOGIC .....	47
3.1. An invertible classical calculus .....	48
3.2. Admissibility of structural rules .....	53
3.3. Completeness .....	58
Notes to Chapter 3 .....	60
4. THE QUANTIFIERS .....	61
4.1. Quantifiers in natural deduction and in sequent calculus .....	61
4.2. Admissibility of structural rules .....	70

4.3. Applications of cut elimination .....	76
4.4. Completeness of classical predicate logic .....	81
Notes to Chapter 4 .....	86
<b>5. VARIANTS OF SEQUENT CALCULI .....</b>	<b>87</b>
5.1. Sequent calculi with independent contexts .....	87
5.2. Sequent calculi in natural deduction style .....	98
5.3. An intuitionistic multisuccedent calculus .....	108
5.4. A classical single succedent calculus .....	114
5.5. A terminating intuitionistic calculus .....	122
Notes to Chapter 5 .....	124
<b>6. STRUCTURAL PROOF ANALYSIS OF AXIOMATIC THEORIES .....</b>	<b>126</b>
6.1. From axioms to rules .....	126
6.2. Admissibility of structural rules .....	131
6.3. Four approaches to extension by axioms .....	134
6.4. Properties of cut-free derivations .....	136
6.5. Predicate logic with equality .....	138
6.6. Application to axiomatic systems .....	141
Notes to Chapter 6 .....	154
<b>7. INTERMEDIATE LOGICAL SYSTEMS .....</b>	<b>156</b>
7.1. A sequent calculus for the weak law of excluded middle .....	157
7.2. A sequent calculus for stable logic .....	158
7.3. Sequent calculi for Dummett logic .....	160
Notes to Chapter 7 .....	164
<b>8. BACK TO NATURAL DEDUCTION .....</b>	<b>165</b>
8.1. Natural deduction with general elimination rules .....	166
8.2. Translation from sequent calculus to natural deduction .....	172
8.3. Translation from natural deduction to sequent calculus .....	179
8.4. Derivations with cuts and non-normal derivations .....	185
8.5. The structure of normal derivations .....	189
8.6. Classical natural deduction for propositional logic .....	202
Notes to Chapter 8 .....	208
<b>CONCLUSION: DIVERSITY AND UNITY IN STRUCTURAL PROOF THEORY .....</b>	<b>211</b>
Comparing sequent calculus and natural deduction .....	211
A uniform logical calculus .....	213
<b>APPENDIX A: SIMPLE TYPE THEORY AND CATEGORIAL GRAMMAR .....</b>	<b>219</b>
A.1. Simple type theory .....	219
A.2. Categorial grammar for logical languages .....	221
Notes to Appendix A .....	224

APPENDIX B: PROOF THEORY AND CONSTRUCTIVE TYPE THEORY .....	225
B.1. Lower-level type theory .....	225
B.2. Higher-level type theory .....	230
B.3. Type systems .....	232
Notes to Appendix B .....	234
APPENDIX C: PESCA – A PROOF EDITOR FOR SEQUENT CALCULUS .....	235
(by Aarne Ranta)	
C.1. Introduction .....	235
C.2. Two example sessions .....	236
C.3. Some commands .....	239
C.4. Axiom files .....	241
C.5. On the implementation .....	242
Notes to Appendix C .....	243
Electronic references .....	243
BIBLIOGRAPHY .....	245
AUTHOR INDEX .....	251
SUBJECT INDEX .....	253
INDEX OF LOGICAL SYSTEMS .....	257