

## CONTENTS

<i>Editors' Preface</i>	page ix
<i>Acknowledgments</i>	xii
<i>Author's Introduction</i>	i
CHAPTER I	
1 A Problem and a Conjecture	6
2 A Proof	7
3 Criticism of the Proof by Counterexamples which are Local but not Global	10
4 Criticism of the Conjecture by Global Counterexamples	13
a. Rejection of the conjecture. The method of surrender	13
b. Rejection of the counterexample. The method of monster-barring	14
c. Improving the conjecture by exception-barring methods. Piece-meal exclusions. Strategic withdrawal or playing for safety	24
d. The method of monster-adjustment	30
e. Improving the conjecture by the method of lemma-incorporation. Proof-generated theorem versus naive conjecture	33
5 Criticism of the Proof-Analysis by Counterexamples which are Global but not Local. The Problem of Rigour	42
a. Monster-barring in defence of the theorem	42
b. Hidden lemmas	43
c. The method of proof and refutations	47
d. Proof versus proof-analysis. The relativisation of the concepts of theorem and rigour in proof-analysis	50
6 Return to Criticism of the Proof by Counterexamples which are Local but not Global. The Problem of Content	57
a. Increasing content by deeper proofs	57
b. Drive towards final proofs and corresponding sufficient and necessary conditions	63
c. Different proofs yield different theorems	65

PROOFS AND REFUTATIONS

7	The Problem of Content Revisited	<i>page</i> 66
	a. The naiveté of the naive conjecture	66
	b. Induction as the basis of the method of proofs and refutations	68
	c. Deductive guessing versus naive guessing	70
	d. Increasing content by deductive guessing	76
	e. Logical versus heuristic counterexamples	82
8	Concept-Formation	83
	a. Refutation by concept-stretching. A reappraisal of monster-barring – and of the concepts of error and refutation	83
	b. Proof-generated versus naive concepts. Theoretical versus naive classification	88
	c. Logical and heuristic refutations revisited	92
	d. Theoretical versus naive concept-stretching. Continuous versus critical growth	93
	e. The limits of the increase in content. Theoretical versus naive refutations	96
9	How Criticism may turn Mathematical Truth into Logical Truth	99
	a. Unlimited concept-stretching destroys meaning and truth	99
	b. Mitigated concept-stretching may turn mathematical truth into logical truth	102

CHAPTER 2

	Editors' Introduction	106
1	Translation of the Conjecture into the 'Perfectly Known' Terms of Vector-Algebra. The Problem of Translation	106
2	Another Proof of the Conjecture	116
3	Some Doubts about the Finality of the Proof. Translation Procedure and the Essentialist versus the Nominalist Approach to Definitions	119

APPENDIX 1

	Another Case-Study in the Method of Proofs and Refutations	
1	Cauchy's Defence of the 'Principle of Continuity'	127
2	Seidel's Proof and the Proof-Generated Concept of Uniform Convergence	131
3	Abel's Exception-Barring Method	133

## CONTENTS

4	Obstacles in the Way of the Discovery of the Method of Proof-Analysis	page 136
APPENDIX 2		
The Deductivist versus the Heuristic Approach		
1	The Deductivist Approach	142
2	The Heuristic Approach. Proof-Generated Concepts	144
	a. Uniform convergence	144
	b. Bounded variation	146
	c. The Carathéodory definition of measurable set	152
	<i>Bibliography</i>	155
	<i>Index of Names</i>	167
	<i>Index of Subjects</i>	170