A logic for "causal modalities"

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Abstract

The analysis of modal vocabulary is one of the main topics of Sellars's early essays. Brandom [3] convincingly argues that the theme of modality is but onec of the battlefields over which Sellars conducted his many-sided attack against *traditional* Empiricism, where Sellars [8] consitutes his more glorious and famous results. He conjectures that it was just by bracketing the analysis of modality that Sellars could eventually end up writing *EPM*, and that he actually attempted to deal with modality in the other big essay of those years, Sellars [9]. This will be my main concern here.

Brandom depicts the structure of the sellarsian campaign in three theses. The first one is a form of *inferentialism*, according to which semantics has to be primarily accounted for not in terms of word-world correspondence relations, but in terms of the functional role that expressions have in language-entry, intra-language and language-exit transitions. The second one is a form of *pragmatism*, according to which these functional roles must be defined in terms of the practices a speaker must engage in and of the abilities she must deploy in order for her usage of expressions to be treated as meaningful. The third one is a *normative* account of semantic vocabulary, according to which the practice of making contents explicit is not only the practice of describing the world as accurately as possible, but also, essentially, the practice of deploying counterfactual reasoning, the practice of saying what might be the case if things were such and such.

Notice that while the first two theses are the well known pillars supporting the building of Brandom [1], the third one is the precious seed sown in Lecture IV of Brandom [2], whose offshoot is the formal semantics system introduced in collaboration with Alp Aker in Lecture V, *Incompatibility Semantics* (*IS*). The fruits of such a semantics should convey all the virtues of these sellarsian insights in an assimilable form for logical and philosophical investigation. And yet there are two main reasons for dissatisfaction with Brandom's *Incompatibility Semantics*.

The first, techincal one is that it is stuck in **S5** modality. The solution to this problem has already been proposed by Göcke et al. [5] and by Peregrin [7]: it consists in the introduction of possible worlds into *IS* as maximally *coherent* sets,

$$PW_{Inc} =_{Def} \{S \mid S \notin Inc \text{ and } \forall X(X \cup S \notin Inc \Rightarrow X \subseteq S)\},$$

and in the definition of an accessibility relation R as 'second-level' compatibility among these sets,

$$w_2 R w_1 \text{ iff } \forall p(w_2 \vDash p \Rightarrow \exists X(X \subseteq w_1 \wedge X \cup p \notin Inc)).$$

That reproduces the familiar gist of a kripkean modal semantics. However, a remark is in order here about the relation between frames and possible worlds. It is common knowledge that in standard truthfunctional semantics models can be equivalently defined in terms of possible worlds or in term of valuation functions. However, as Aker and Peregrin notice, *satisfiability* in a model in *IS* corresponds to *coherence* in a model. Now, in *IS* it is possible to define models for s in terms of possible worlds

$$M_s = \{w \in PW_{Inc} \mid w \vDash_{Inc} s\}.$$

But it makes no sense to select such a set in terms of different *incoherence relations*. Indeed, Aker shows how to define an *incoherence frame* in terms of a maximally coherent set of sentences Z, but it's not possible conversely to define a single possible world Z in terms of an *incoherence frame*: as it's easy to see, if $p \cup q \in Inc$ then either p but not q, or q but not p can be pushed into Z.

The second reason for dissatisfaction digs deeper in the interpretation of material inferences and directly puts into question the adequacy of *IS* as a semantic representation of the third of the above sellarsian theses. In fact the main logical trail of Sellars [9] is the idea that the inferences which make explicit conceptual contents must be both *modally robust* and criterially *defeasible*: that is clearly the kernel of Sellars's germinal notion of "causal modality". This feature is explicitly acknowledged by Brandom himself when he points out that material inferences are nonmonotonic. And yet his incompatibility-entailment is fully monotonic.

I try to solve this clash by applying to *IS* some standard results from nonmonotonic logics for *defeasible reasoning*. Indeed, once the notion of possible world has been introduced, it's quite plain to develop in *IS* the definition of a *Preferential Semantics* (Shoham [10]) suitable to represent *defeasible reasoning* as characterized by *GM* conditions (Gabbay [4], Makinson [6]). There's just one major twist. Generally speaking, a particularly effective way to introduce *Preferential Semantics* is to talk about semantic valuation functions. Thus, given the set V of possible valuations over a language $\mathcal{L}$, if we consider a subset $W \subseteq V$ and an order relation $\prec$ on W, then the poset $(W, \prec)$ is a *preferential model* which can represent the degree of normality of any given inference. Then for any set of premises X we can define $X \mid\!\sim_{\prec} Y$ if and only if Y follows from X in all minimal models for X according to $\prec$. However there's not really anything like valuation functions in *IS*, and a whole articulation of possible worlds corresponds to each *incoherence frame*. In this sense to establish an order on *incoherence relations*, i.e. $\overrightarrow{Inc}$: $Inc_1, Inc_2, \ldots, Inc_n$, simply wouldn't do because models are specified not by Incs but by possible worlds. So, in *IS*, we have to define an order on possible worlds. More specifically, given a subset $W \subseteq PW_{Inc}$ and a set of premises X, let $|X|_{W_{Inc}}$ be the set of models for X in W_{Inc}, i.e. $|X|_{W_{Inc}} = \{w \in W_{Inc} \mid w \vDash_{Inc} X\}$, and let $min_{\prec_{W_{Inc}}} |X|_{W_{Inc}}$ the set of minimal models for X in $\prec$. Now it's easy to define a nonmonotonic incompatibility entailment:

$$X \mid\!\sim_{\prec_{W_{Inc}}} Y \text{ iff } \bigcap_{p \in Y} I(\{p\}) \subseteq I(min_{\prec_{W_{Inc}}} |X|_{W_{Inc}})$$

Notice that, as a consequence of the lack of correlation between *incoherence frames* and models, the whole preferential ordering lies inside one single *incoherence frame*. So, in this representation of defeasible reasoning, the revision of conceptual contents doesn't affect their underlying – say, *implicit* – definition represented by the *incoherence relation*. In Brandom's framework, this feature naturally suggests an *idealistic* interpretation of the logic of the process of making conceptual contents explicit. The question is still open whether such an interpretation is adequate to Sellars's account.

References

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